Sequences and series - Practice test 1

Name: _____ Date: _____

10 questions | 53 marks | 70 minutes

Terms, sums and first model choices.

A scientific calculator may be used.

1. $u_1 = 4$ and $u_{n+1} = u_n + 7$. [5 marks]

(a) Find u_2, u_3, u_4. [2 marks]

(b) Give an explicit rule and find u_{30}. [3 marks]

2. An arithmetic sequence has $u_3 = 16, u_9 = 46$. [5 marks]

(b) Decide whether 121 is a term. [3 marks]

3. An auditorium has 8 seats in its first row and 3 extra seats in each next row. [5 marks]

(a) Find the seats in row 12. [2 marks]

(b) Find the total seats in 12 rows. [3 marks]

4. The geometric sequence begins $8, -24, 72, \ldots$ [5 marks]

(a) Find its ratio and seventh term. [2 marks]

(b) Find the sum of its first six terms. [3 marks]

5. A geometric sequence has $u_1 = 5, u_3 = 20$. [5 marks]

(a) Find all possible real ratios. [2 marks]

(b) For each ratio, find u_2 and u_4. [3 marks]

6. A ball falls 3 m and rebounds to one-third of the preceding height after every bounce. [6 marks]

(a) Find the first two rebound heights. [2 marks]

(b) Find its ideal total vertical distance. [4 marks]

7. A flushing process starts with 10 g of salt. Each step retains 80%, then adds 6 g. [6 marks]

(a) Write a recurrence and calculate the amount after three steps. [3 marks]

(b) Find and interpret the equilibrium. [3 marks]

8. AUD 300 is deposited at the end of each year for four years at 6% annual interest. [5 marks]

(a) Write the four contributions to the final balance. [2 marks]

(b) Calculate the balance immediately after the fourth deposit. [3 marks]

9. Consider $3 + 5 + \cdots + (2n + 1) = n(n + 2)$. [6 marks]

(a) Check the base case and state a valid induction hypothesis. [2 marks]

(b) Complete the induction step and conclusion. [4 marks]

10. A student says: “If sequence terms approach zero, their sum must approach zero.” [5 marks]

(a) Use $1, 1/2, 1/4, \ldots$ to refute the statement. [2 marks]

(b) Explain the distinction between a term and a partial sum. [3 marks]