Sequences and series - Practice test 2

Name: _____ Date: _____

10 questions | 56 marks | 70 minutes

Comparing rules, timing and proof structure.

A scientific calculator may be used.

1. Two lists are $A: 2, 5, 8, 11, \ldots$ and $B: 2, 6, 18, 54, \ldots$. [5 marks]

(a) Classify each and give its explicit rule. [3 marks]

(b) Find the fourth-term sum for each. [2 marks]

2. An arithmetic sequence has $u_3 = 11, u_8 = 31$. [5 marks]

(a) Find its rule. [2 marks]

(b) Find the first term greater than 100 and its index. [3 marks]

3. Four arithmetic means are inserted between 7 and 32. [4 marks]

(a) Find the means. [2 marks]

(b) Find the sum of all six terms. [2 marks]

4. A positive geometric sequence has $u_2 = 12, u_5 = 96$. [5 marks]

(a) Find a and r. [2 marks]

(b) Find the sum of its first five terms. [3 marks]

5. Consider $4 - 2 + 1 - 1/2 + \cdots$. [6 marks]

(a) Find its first six-term sum. [3 marks]

(b) Find the infinite sum and compare it with the six-term sum. [3 marks]

6. The geometric series $1 + 1/3 + 1/9 + \cdots$ is approximated by its first n terms. [6 marks]

(a) Find its infinite sum and tail after n terms. [3 marks]

(b) Find the least n for a tail below 0.001. [3 marks]

7. A debt is AUD 1500 initially. Each month 2% interest is added, then AUD 200 is paid. [6 marks]

(a) Calculate the first two updated balances. [2 marks]

(b) Find a closed formula valid before the debt is repaid. [4 marks]

8. A painter covers 6 square metres on day 1 and 2 extra square metres each later day. [6 marks]

(a) Write the total area after n days. [3 marks]

(b) Find the first day by which the total exceeds 100 square metres. [3 marks]

9. For fixed real a, d, let $S_n = a + (a + d) + \cdots + [a + (n - 1)d]$. [7 marks]

(a) State the arithmetic-sum identity and check $n = 1$. [2 marks]

(b) Prove the identity by induction. [5 marks]

10. An account begins at AUD 500. Compare model $A_{n+1} = 1.1A_n + 50$ with model $C_{n+1} = 1.1(C_n + 50)$. [6 marks]

(a) Explain the timing difference. [2 marks]

(b) Find both balances after two periods and their difference. [4 marks]