Sequences and series - Practice test 4

Name: _____ Date: _____

10 questions | 55 marks | 70 minutes

Applications, counterexamples and model comparisons.

A scientific calculator may be used.

1. A display is built in rows containing $4, 7, 10, \ldots$ boxes. [5 marks]

(a) Find the boxes in row 20. [2 marks]

(b) How many boxes are needed for the first 20 rows? [3 marks]

2. A technician measures successive pulses $12, -6, 3, -1.5, \ldots$ in arbitrary signed units. [6 marks]

(a) Find the eight-pulse total. [3 marks]

(b) Find the limiting total and tail after eight pulses. [3 marks]

3. After a treatment, a stock retains 70% and then gains 12 units. It starts at 10 units. [6 marks]

(a) Find a recurrence and its first two updated values. [2 marks]

(b) Find an explicit rule and the long-term stock. [4 marks]

4. AUD 250 is deposited three times, once per year, at 4% annual interest. [6 marks]

(a) Find the value after three years if each deposit is made at the end of a year. [3 marks]

(b) Find the value if deposits are made at the start of each year, and explain the difference. [3 marks]

5. A AUD 20,000 machine can be modelled by losing AUD 2000 yearly or retaining 90% yearly. [6 marks]

(a) Find both values after three years. [2 marks]

(b) Which model predicts the higher value after three years, and by how much? [2 marks]

(c) Explain the different mathematical mechanisms. [2 marks]

6. A geometric sequence has first term 2 and ratio 3. Let S_n be its first n-term sum. [7 marks]

(a) Find an explicit expression for S_n. [2 marks]

(b) Prove the result by induction. [5 marks]

7. The recurrence $u_{n+1} = u_n + 4$ is given without an initial value. [4 marks]

(a) Choose the initial value that makes $u_5 = 30$. [2 marks]

(b) Can the same recurrence also produce $u_5 = 10$? Explain. [2 marks]

8. A culture has 500 organisms initially and grows by 25% per day in a model. [5 marks]

(a) Give the term rule at whole-number day $n \geq 0$. [2 marks]

(b) Find the first whole day with more than 2000 organisms. [3 marks]

9. Consider the proposed proof: "Assume $S_{k+1} = (k+1)^2$. Therefore the sum of the first $k+1$ odd numbers is $(k+1)^2$." [5 marks]

(a) Explain why this is not an induction step. [2 marks]

(b) Replace it with a valid step. [3 marks]

10. Two sequences start $1, 2, 3$. One continues with $u_n = n$; another uses $v_n = n + (n-1)(n-2)(n-3)$. [5 marks]

(a) Check that both rules match the first three terms. [2 marks]

(b) Find the fourth terms and explain what this shows about guessing patterns. [3 marks]