Sequences and series - Practice test 5

Name: _____ Date: _____

10 questions | 62 marks | 75 minutes

Integrated sequence reasoning and more demanding proofs.

A scientific calculator may be used.

1. An arithmetic sequence satisfies $u_2 + u_6 = 30$ and $S_{10} = 210$. [6 marks]

(a) Find the first term and common difference. [4 marks]

(b) Find u_{20}. [2 marks]

2. Three positive geometric terms have sum 26 and product 216. [7 marks]

(a) Show that the middle term is 6. [3 marks]

(b) Find all possible ordered triples. [4 marks]

3. A convergent geometric series has infinite sum 16 and its first two terms sum to 12. [7 marks]

(a) Find all real common ratios. [4 marks]

(b) Find the corresponding first terms and explain why both converge. [3 marks]

4. $u_0 = 2$ and $u_{n+1} = u_n^2$. [5 marks]

(a) Find u_1, u_2, u_3. [2 marks]

(b) Give an explicit rule and explain why this is not an ordinary geometric sequence. [3 marks]

5. A conservation model begins at 50 animals and updates by retaining 90% then adding 8 animals. [6 marks]

(a) Find its explicit rule and value after two steps. [3 marks]

(b) Explain why 80 is never attained at a finite step in this model. [3 marks]

6. End-of-year deposits are AUD 100, AUD 110 and AUD 121 in years 1, 2 and 3. Interest is 10% yearly. [5 marks]

(a) Find the three contributions immediately after the third deposit. [2 marks]

(b) Find the balance and explain why the usual equal-deposit formula cannot be applied directly. [3 marks]

7. A ball drops 5 m and rebounds to 60% of the previous height each time. [7 marks]

(a) Find the ideal total vertical distance. [3 marks]

(b) Find the total distance by the time it has returned to the ground after its second rebound. [2 marks]

(c) Find the remaining distance in the infinite model. [2 marks]

8. For fixed a and $r \neq 1$, prove $S_n = a(1 - r^n)/(1 - r)$. [7 marks]

(a) Check the base and state the hypothesis. [2 marks]

(b) Prove the next case and state how $r = 1$ is handled. [5 marks]

9. A proposed identity is $1 + 2 + \cdots + n = n(n + 1)/2 + 7$. [6 marks]

(a) Check its base at $n = 1$. [2 marks]

(b) Show that assuming its k case would nevertheless lead formally to the same wrong $k + 1$ formula. Explain the lesson. [4 marks]

10. A model uses $u_n = 100(1.04)^n$ at whole-number years $n \geq 0$. [6 marks]

(a) Find the first whole year when $u_n > 200$. [3 marks]

(b) Explain why rounding the real solution of $100(1.04)^n = 200$ to the nearest integer is not the correct general method. [3 marks]