Why the imaginary unit is needed

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. State why $x^2 = -5$ has no real solution. [2 marks]

For $x \geq 0$ or $x < 0$, multiplying x by itself gives a non-negative result.

Answer: The square of every real number is non-negative.

2. Simplify the principal square root $\sqrt{-36}$. [2 marks]

$\sqrt{-36} = i\sqrt{36} = 6i$.

Answer: $6i$.

3. Solve $z^2 + 81 = 0$. [2 marks]

$z^2 = -81$.

Both $9i$ and $-9i$ square to -81.

Answer: $z = \pm 9i$.

4. Solve $4z^2 + 64 = 0$. [2 marks]

$4z^2 = -64$, hence $z^2 = -16$.

Take both complex roots.

Answer: $z = \pm 4i$.

5. Evaluate $(3i)^2$ and explain why the answer is real. [2 marks]

$(3i)^2 = 9i^2 = 9(-1) = -9$.

The factor i^2 is real.

Answer: -9.

6. Simplify $\sqrt{-20}$ exactly, then check its square. [3 marks]

$\sqrt{-20} = i\sqrt{4 \times 5} = 2i\sqrt{5}$.

$(2i\sqrt{5})^2 = 4(-1)5 = -20$.

Answer: $2i\sqrt{5}$; its square is -20.