Conjugates and division

6 questions | 17 marks

Equivalent correct methods earn the corresponding marks.

1. Find the conjugate of $-5-2i$. [2 marks]

Keep the real part and reverse the imaginary component.

Answer: $-5+2i$.

2. Find $z\bar{z}$ for $z=4-3i$. [2 marks]

$z\bar{z}=4^2+(-3)^2=25$.

Answer: 25.

3. Simplify $\frac{2-i}{3+i}$. [3 marks]

Multiply by $(3-i)/(3-i)$.

The numerator is $6-2i-3i+i^2=5-5i$ and denominator is 10.

Answer: $\frac{1}{2}-\frac{1}{2}i$.

4. Simplify $\frac{5}{1+2i}$. [3 marks]

Multiply by $(1-2i)/(1-2i)$.

The denominator is 5, giving $5(1-2i)/5$.

Answer: $1-2i$.

5. Solve $(2-i)z=7+i$. [3 marks]

$z=(7+i)/(2-i)$.

Multiply by $(2+i)/(2+i)$: the numerator is $13+9i$ and denominator 5.

Answer: $z=\frac{13}{5}+\frac{9}{5}i$.

6. Show that $\frac{z}{\bar{z}}$ has real value only when $ab=0$, where $z=a+bi\neq 0$ and a, b are real. [4 marks]

$z/\bar{z}=(a+bi)^2/(a^2+b^2)$.

The numerator is a^2-b^2+2abi.

The denominator is positive, so its imaginary part is zero exactly when $2ab=0$.

Answer: The imaginary part is $\frac{2ab}{a^2+b^2}$, so it vanishes exactly when $ab=0$.