Quadratic equations with non-real roots

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $z^2 + 25 = 0$. [2 marks]

$z^2 = -25$, whose two roots are $\pm 5i$.

Answer: $z = \pm 5i$.

2. Solve $z^2 - 4z + 8 = 0$. [3 marks]

Complete the square: $(z - 2)^2 = -4$.

Answer: $z = 2 \pm 2i$.

3. Solve $3z^2 + 12 = 0$. [2 marks]

Divide by 3 to obtain $z^2 = -4$.

Answer: $z = \pm 2i$.

4. Solve $z^2 + 6z + 11 = 0$ exactly. [3 marks]

$(z + 3)^2 = -2$.

Take both roots and subtract 3.

Answer: $z = -3 \pm i\sqrt{2}$.

5. Check directly that $1 + 3i$ solves $z^2 - 2z + 10 = 0$. [3 marks]

$(1 + 3i)^2 = -8 + 6i$.

$(-8 + 6i) - 2(1 + 3i) + 10 = 0 + 0i$.

Answer: The substitution gives zero.

6. For real k, determine when $z^2 + 4z + k = 0$ has non-real roots. [3 marks]

The discriminant is $16 - 4k$.

It is negative exactly when $16 - 4k < 0$, or $k > 4$.

Answer: $k > 4$.