Conjugate roots of real quadratics

6 questions | 17 marks

Equivalent correct methods earn the corresponding marks.

1. A real quadratic has root $-3+5i$. State the other root. [2 marks]

Use the conjugate-root property.

Answer: $-3-5i$.

2. Find the monic real quadratic with roots $1 \pm 4i$. [3 marks]

The sum is 2 and product is $1^2+4^2=17$.

Answer: $z^2-2z+17=0$.

3. Find the monic real quadratic with roots $-2 \pm i$. [3 marks]

Expand $(z+2-i)(z+2+i)=(z+2)^2+1$.

Answer: $z^2+4z+5=0$.

4. One root of $2z^2+bz+c=0$ is $3+i$. Find real b, c. [3 marks]

The monic quadratic is $z^2-6z+10$.

Multiply by the required leading coefficient 2.

Answer: $b=-12$, $c=20$.

5. A student claims a real quadratic has exactly the roots $2+i$ and $3-i$. Explain why this is impossible. [3 marks]

$(2+i)(3-i)=7+i$.

A real quadratic's product of roots must be real.

Answer: The two roots are not conjugates; their sum is 5, but their product is $7+i$, which is not real.

6. For $p(z)=z-(1+i)$, test whether the conjugate of its root is also a root. Explain the result. [3 marks]

The coefficient $-(1+i)$ is non-real.

The real-coefficient condition is not satisfied.

Answer: $1-i$ is not a root because $p(1-i)=-2i$.