Linear factorisation over the complex numbers

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

1. Factor $z^2 + 16$ into complex linear factors. [2 marks]

The roots satisfy $z^2 = -16$ and are $\pm 4i$.

Answer: $(z - 4i)(z + 4i)$.

2. Factor $z^2 - 2z + 5$ over the complex numbers. [3 marks]

$(z - 1)^2 = -4$ gives roots $1 \pm 2i$.

Answer: $(z - 1 - 2i)(z - 1 + 2i)$.

3. Factor $3z^2 + 12$ over the complex numbers. [3 marks]

First take out 3; $z^2 + 4$ has roots $\pm 2i$.

Answer: $3(z - 2i)(z + 2i)$.

4. Expand $(z + 2 - 3i)(z + 2 + 3i)$. [3 marks]

Use difference of squares: $(z + 2)^2 - (3i)^2 = (z + 2)^2 + 9$.

Answer: $z^2 + 4z + 13$.

5. A quadratic has leading coefficient -2 and roots $2 \pm i$. Write it in factored and expanded forms. [4 marks]

The product of conjugate factors is $(z - 2)^2 + 1$.

Multiply all coefficients by -2.

Answer: $-2(z - 2 - i)(z - 2 + i) = -2z^2 + 8z - 10$.

6. Factor $(z^2 + 1)(z^2 - 4)$ into four linear factors and list its roots. [4 marks]

The first quadratic has complex roots $\pm i$.

The second has real roots ± 2.

Each root supplies one factor.

Answer: $(z - i)(z + i)(z - 2)(z + 2)$; roots $i, -i, 2, -2$.