Complex numbers - practice test 1

10 questions | 50 marks | 65 minutes

Build from Cartesian components and arithmetic to quadratic roots, geometric distances and a short identity proof.

Equivalent correct methods earn the corresponding marks.

1. Let $z = -3 + 4i$. [5 marks]

(a) State $\text{Re}(z)$ and $\text{Im}(z)$. [2 marks]

Compare with $a + bi$; the imaginary part is the coefficient of i.

Answer: -3 and 4.

(b) Find $\overline{z}$ and $z\overline{z}$. [3 marks]

Reverse the imaginary component.

$z\overline{z} = (-3)^2 + 4^2 = 25$.

Answer: $-3 - 4i$ and 25.

2. Use $i^2 = -1$. [5 marks]

(a) Simplify i^{31}. [2 marks]

$31 = 4(7) + 3$, so $i^{31} = i^3 = -i$.

Answer: $-i$.

(b) Calculate $(2 + i)^2 - (1 - 2i)$. [3 marks]

$(2 + i)^2 = 3 + 4i$.

Subtract $1 - 2i$ to obtain $2 + 6i$.

Answer: $2 + 6i$.

3. Let $u = 3 - 2i$ and $v = 1 + 4i$. [5 marks]

(a) Find $u + v$ and $u - v$. [2 marks]

Add/subtract matching components.

Answer: $4 + 2i$ and $2 - 6i$.

(b) Find uv in Cartesian form. [3 marks]

Expand to $3 + 12i - 2i - 8i^2$.

Use $i^2 = -1$.

Answer: $11 + 10i$.

4. Consider $w = \frac{4+3i}{2-i}$. [5 marks]

(a) Write w in Cartesian form. [3 marks]

Multiply by $(2 + i)/(2 + i)$.

Numerator $= 5 + 10i$; denominator $= 5$.

Answer: $1 + 2i$.

(b) Check your result by multiplication. [2 marks]

Expand: $2 + 4i - i - 2i^2 = 4 + 3i$.

Answer: $(2 - i)(1 + 2i) = 4 + 3i$.

5. Consider $p(z) = z^2 - 2z + 10$. [5 marks]

(a) Solve $p(z) = 0$ over the complex numbers. [3 marks]

Complete the square: $(z - 1)^2 = -9$.

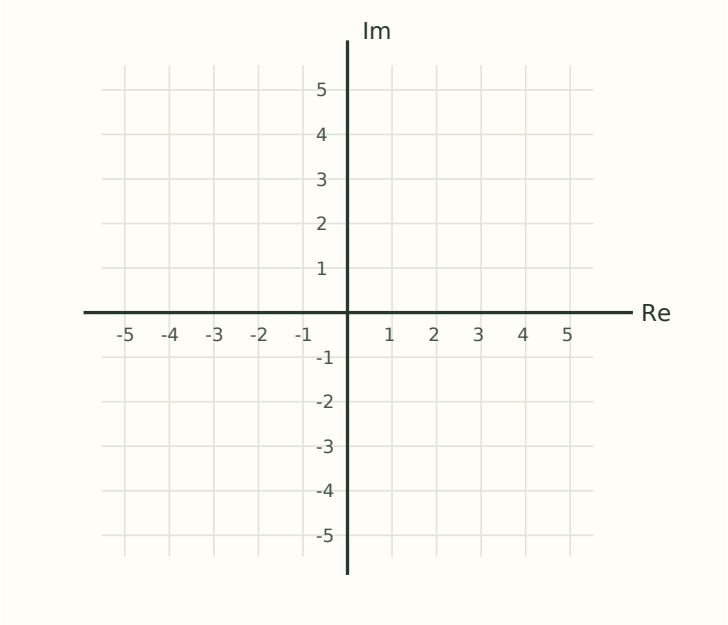
Answer: $z = 1 \pm 3i$.

(b) Write $p(z)$ as a product of complex linear factors. [2 marks]

Use one factor for each root.

Answer: $(z - 1 - 3i)(z - 1 + 3i)$.

6. On the supplied Argand grid, A represents $-2+i$ and B represents $2+4i$. [5 marks]



(a) Plot and label A and B. [2 marks]

Use real horizontal and imaginary vertical coordinates.

Answer: $A=(-2,1)$ and $B=(2,4)$.

(b) Find the displacement from A to B and the length AB. [3 marks]

$B-A=4+3i$.

$AB=\sqrt{4^2+3^2}=5$.

Answer: $4+3i$; $AB=5$.

7. A monic quadratic with real coefficients has a root $-2+3i$. [5 marks]

(a) State its other root. [1 mark]

Non-real roots of a real quadratic are conjugate.

Answer: $-2-3i$.

(b) Find the quadratic and verify its constant term. [4 marks]

Expand $(z+2-3i)(z+2+3i)=(z+2)^2+9$.

The product of the roots is $(-2)^2+3^2=13$.

Answer: $z^2+4z+13$; constant term 13.

8. Real numbers x, y satisfy $(x+1)+(2y-3)i = 5+7i$. [5 marks]

(a) Find x and y. [3 marks]

$x+1=5$ and $2y-3=7$.

Answer: $x=4$, $y=5$.

(b) Use your values to find $|x+yi|$. [2 marks]

$|4+5i| = \sqrt{16+25}$.

Answer: $\sqrt{41}$.

9. A position is recorded by a complex number measured in metres. A robot starts at $3-i$ and moves in a straight line with displacement $-5+4i$. [5 marks]

(a) Find its final position. [2 marks]

Add starting position and displacement.

Answer: $-2+3i$ metres.

(b) Find its distance from the origin and explain why this is not the total distance travelled. [3 marks]

Final distance is $\sqrt{(-2)^2+3^2}$.

The travel distance is the displacement length $\sqrt{41}$ metres; these measure different quantities.

Answer: $\sqrt{13}$ metres.

10. Let $z=a+bi$, where a, b are real. [5 marks]

(a) Show that $z+\overline{z}$ is real. [2 marks]

Add $(a+bi)+(a-bi)$; imaginary terms cancel.

Answer: $z+\overline{z}=2a$.

(b) Prove $|z|^2 = z\overline{z}$. [3 marks]

By definition, $|z|^2 = a^2+b^2$.

Expand $(a+bi)(a-bi) = a^2-b^2i^2 = a^2+b^2$.

Answer: Both equal a^2+b^2.