Complex numbers - practice test 1

Name: _____ Date: _____

10 questions | 50 marks | 65 minutes

Build from Cartesian components and arithmetic to quadratic roots, geometric distances and a short identity proof.

No calculator needed.

1. Let $z = -3 + 4i$. [5 marks]

(a) State $\text{Re}(z)$ and $\text{Im}(z)$. [2 marks]

(b) Find $\overline{z}$ and $z\overline{z}$. [3 marks]

2. Use $i^2 = -1$. [5 marks]

(a) Simplify i^{31}. [2 marks]

(b) Calculate $(2 + i)^2 - (1 - 2i)$. [3 marks]

3. Let $u = 3 - 2i$ and $v = 1 + 4i$. [5 marks]

(a) Find $u + v$ and $u - v$. [2 marks]

(b) Find uv in Cartesian form. [3 marks]

4. Consider $w = \frac{4+3i}{2-i}$. [5 marks]

(a) Write w in Cartesian form. [3 marks]

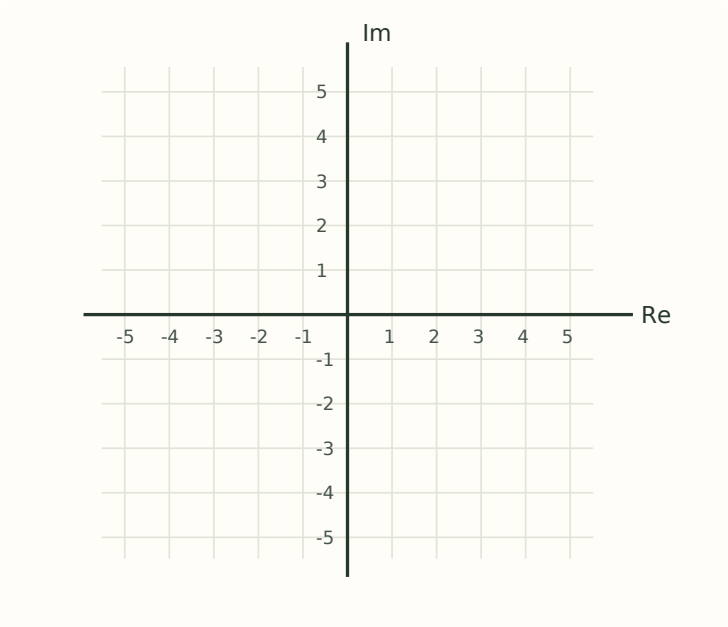
(b) Check your result by multiplication. [2 marks]

5. Consider $p(z) = z^2 - 2z + 10$. [5 marks]

(a) Solve $p(z) = 0$ over the complex numbers. [3 marks]

(b) Write $p(z)$ as a product of complex linear factors. [2 marks]

6. On the supplied Argand grid, A represents $-2+i$ and B represents $2+4i$. [5 marks]



(a) Plot and label A and B. [2 marks]

(b) Find the displacement from A to B and the length AB. [3 marks]

7. A monic quadratic with real coefficients has a root $-2+3i$. [5 marks]

(a) State its other root. [1 mark]

(b) Find the quadratic and verify its constant term. [4 marks]

8. Real numbers x, y satisfy $(x+1)+(2y-3)i = 5+7i$. [5 marks]

(a) Find x and y. [3 marks]

(b) Use your values to find $|x+yi|$. [2 marks]

9. A position is recorded by a complex number measured in metres. A robot starts at $3 - i$ and moves in a straight line with displacement $-5 + 4i$. [5 marks]

(a) Find its final position. [2 marks]

(b) Find its distance from the origin and explain why this is not the total distance travelled. [3 marks]

10. Let $z = a + bi$, where a, b are real. [5 marks]

(a) Show that $z + \overline{z}$ is real. [2 marks]

(b) Prove $|z|^2 = z\overline{z}$. [3 marks]