Complex numbers - practice test 2

10 questions | 54 marks | 70 minutes

Connect equations and factorisation with Argand geometry, midpoint conditions and real-parameter questions.

Equivalent correct methods earn the corresponding marks.

1. Let $z = 5 - 2i$. [5 marks]

(a) Find iz and $2z - i$. [3 marks]

$iz = 5i - 2i^2 = 2 + 5i$.

$2z - i = 10 - 4i - i$.

Answer: $2 + 5i$ and $10 - 5i$.

(b) Find $\text{Re}(iz)$ and $\text{Im}(2z - i)$. [2 marks]

Read the coefficients in the simplified answers.

Answer: 2 and -5.

2. Consider $A = 2 - 3i$ and $B = -1 - 2i$. [6 marks]

(a) Calculate AB. [3 marks]

$AB = -2 - 4i + 3i + 6i^2 = -8 - i$.

Answer: $-8 - i$.

(b) Find $|AB|$ by using $|A||B|$. [3 marks]

$|A| = \sqrt{13}$ and $|B| = \sqrt{5}$.

Multiply and compare with $|-8 - i| = \sqrt{65}$.

Answer: $\sqrt{65}$.

3. Solve the equation $(1+2i)w = 3-i$. [5 marks]

(a) Find w in Cartesian form. [3 marks]

$w = (3-i)(1-2i)/5$.

The numerator is $1-7i$.

Answer: $w = \frac{1}{5} - \frac{7}{5}i$.

(b) Find $w + \overline{w}$ and $w - \overline{w}$. [2 marks]

Add/subtract conjugates; real parts double in the sum and imaginary terms double in the difference.

Answer: $\frac{2}{5}$ and $-\frac{14}{5}i$.

4. Consider the equation $z^2 + 8z + 20 = 0$. [6 marks]

(a) Solve it by completing the square. [3 marks]

$(z+4)^2 = -4$.

Answer: $z = -4 \pm 2i$.

(b) Verify one root by direct substitution. [3 marks]

$z^2 = 12 - 16i$.

Add $8z = -32 + 16i$ and 20 to obtain zero.

Answer: For $z = -4 + 2i$, the expression is zero.

5. Points A and B represent $-3+2i$ and $5-4i$. [5 marks]

(a) Find their midpoint. [2 marks]

Average each pair of components: $(-3+5)/2=1$, $(2-4)/2=-1$.

Answer: $1-i$.

(b) Find the distance between their conjugate points and explain why it equals AB. [3 marks]

$AB=\sqrt{8^2+(-6)^2}=10$.

Reflection in the real axis preserves distance.

Answer: 10.

6. A complex number z satisfies $|z|=10$ and $\operatorname{Im}(z)=-6$. [5 marks]

(a) Find all possible z. [3 marks]

Write $z=a-6i$.

$a^2+36=100$, so $a=\pm 8$.

Answer: $8-6i$ and $-8-6i$.

(b) Find the distance between these two points. [2 marks]

Their real coordinates differ by 16 and their imaginary coordinates agree.

Answer: 16.

7. A polynomial is $p(z)=(z-2)(z^2+4z+8)$. [5 marks]

(a) List all its complex roots. [3 marks]

Either $z-2=0$ or $(z+2)^2=-4$.

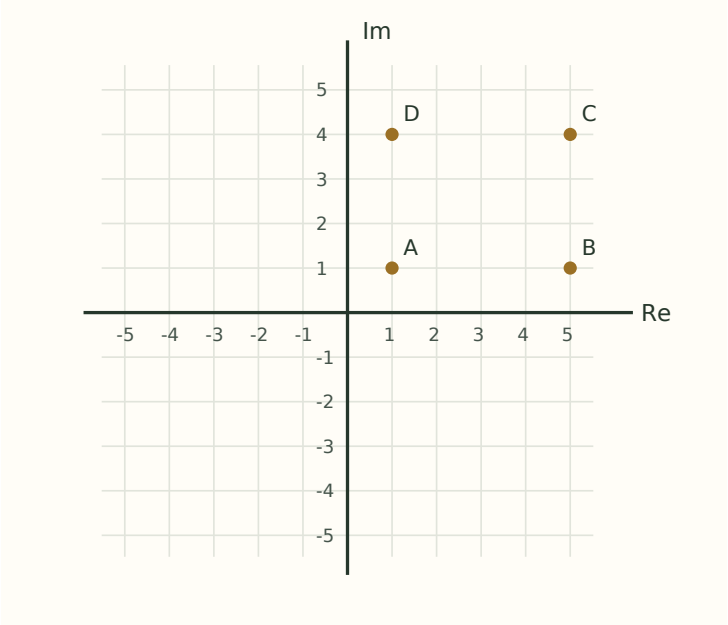
Answer: $2,\ -2+2i,\ -2-2i$.

(b) Give its complete linear factorisation. [2 marks]

Convert the three roots into factors.

Answer: $(z-2)(z+2-2i)(z+2+2i)$.

8. An Argand rectangle has vertices $1+i$, $5+i$, $5+4i$ and $1+4i$. [5 marks]



(a) Find its area and the length of either diagonal. [3 marks]

The side lengths are 4 and 3.

Pythagoras gives diagonal $\sqrt{16+9}=5$.

Answer: Area 12 square units; diagonal length 5.

(b) Find the complex number at its centre. [2 marks]

Take the midpoint of $1+i$ and $5+4i$.

Answer: $3+\frac{5}{2}i$.

9. For real t, define $z = \frac{t+i}{1-i}$. [6 marks]

(a) Express z in terms of t in Cartesian form. [3 marks]

Multiply numerator/denominator by $1 + i$.

The numerator is $t - 1 + (t + 1)i$.

Answer: $z = \frac{t-1}{2} + \frac{t+1}{2}i$.

(b) Find t if z is real, and find the corresponding z. [3 marks]

A real number has imaginary part zero, so $t + 1 = 0$.

Substitute $t = -1$ into the real component.

Answer: $t = -1$ and $z = -1$.

10. A student proposes roots $1 + 2i$ and $1 - 3i$ for a monic real quadratic. [6 marks]

(a) Explain why this proposal cannot be correct. [2 marks]

A real quadratic's non-real roots must be paired as $a + bi, a - bi$.

Answer: The roots are not conjugates.

(b) Expand the factors formed from the proposed roots to confirm the problem. [4 marks]

The sum of proposed roots is $2 - i$ and product is $7 - i$.

The resulting coefficients are not all real.

Answer: $(z - 1 - 2i)(z - 1 + 3i) = z^2 + (-2 + i)z + 7 - i$.