Complex numbers - practice test 2

Name: _____ Date: _____

10 questions | 54 marks | 70 minutes

Connect equations and factorisation with Argand geometry, midpoint conditions and real-parameter questions.

No calculator needed.

1. Let $z = 5 - 2i$. [5 marks]

(a) Find iz and $2z - i$. [3 marks]

(b) Find $\text{Re}(iz)$ and $\text{Im}(2z - i)$. [2 marks]

2. Consider $A = 2 - 3i$ and $B = -1 - 2i$. [6 marks]

(a) Calculate AB. [3 marks]

(b) Find $|AB|$ by using $|A||B|$. [3 marks]

3. Solve the equation $(1 + 2i)w = 3 - i$. [5 marks]

(a) Find w in Cartesian form. [3 marks]

(b) Find $w + \overline{w}$ and $w - \overline{w}$. [2 marks]

4. Consider the equation $z^2 + 8z + 20 = 0$. [6 marks]

(a) Solve it by completing the square. [3 marks]

(b) Verify one root by direct substitution. [3 marks]

5. Points A and B represent $-3 + 2i$ and $5 - 4i$. [5 marks]

(a) Find their midpoint. [2 marks]

(b) Find the distance between their conjugate points and explain why it equals AB. [3 marks]

6. A complex number z satisfies $|z| = 10$ and $\text{Im}(z) = -6$. [5 marks]

(a) Find all possible z. [3 marks]

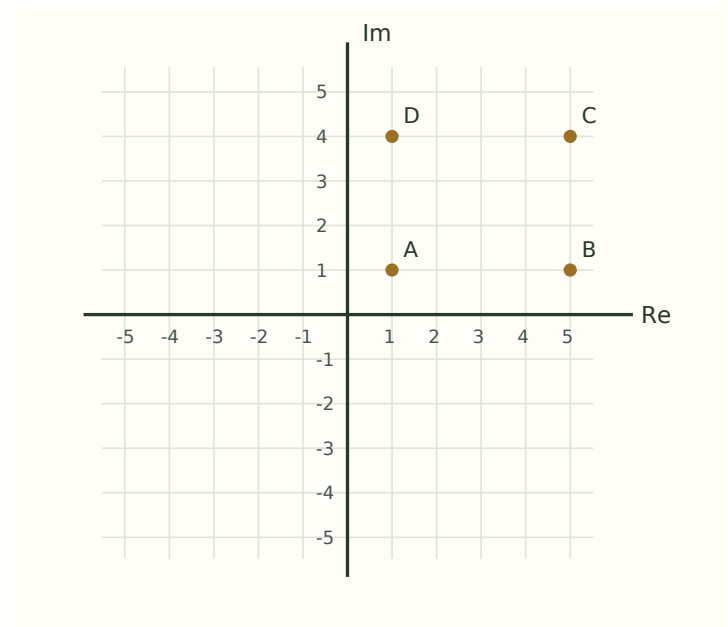
(b) Find the distance between these two points. [2 marks]

7. A polynomial is $p(z) = (z - 2)(z^2 + 4z + 8)$. [5 marks]

(a) List all its complex roots. [3 marks]

(b) Give its complete linear factorisation. [2 marks]

8. An Argand rectangle has vertices $1+i$, $5+i$, $5+4i$ and $1+4i$. [5 marks]



(a) Find its area and the length of either diagonal. [3 marks]

(b) Find the complex number at its centre. [2 marks]

9. For real t, define $z = \frac{t+i}{1-i}$. [6 marks]

(a) Express z in terms of t in Cartesian form. [3 marks]

(b) Find t if z is real, and find the corresponding z. [3 marks]

10. A student proposes roots $1 + 2i$ and $1 - 3i$ for a monic real quadratic. [6 marks]

(a) Explain why this proposal cannot be correct. [2 marks]

(b) Expand the factors formed from the proposed roots to confirm the problem. [4 marks]