# Complex numbers - reasoning test

Name: _____ Date: _____

10 questions | 56 marks | 75 minutes

Use component equations, geometric constraints and short proofs to justify complex-number results.

No calculator needed.

**1.** Let $z = -2 - 3i$. [5 marks]

**(a)** Find $-\overline{z}$ and $\overline{-z}$. [3 marks]

**(b)** Explain why the equality holds for every complex $z$. [2 marks]

**2.** Consider $(2 + i)z = 5$. [5 marks]

**(a)** Solve for $z$. [2 marks]

**(b)** Use moduli to check the magnitude of your result. [3 marks]

**3.** Use the cycle of powers of $i$. [5 marks]

**(a)** Evaluate $i^{25} - i^{14} + i^{8}$. [3 marks]

**(b)** Find $i^{-1}$ and explain why it is not $i$. [2 marks]

**4.** A complex number satisfies $z + 2\overline{z} = 9 - 4i$. [6 marks]

**(a)** Find $z$ by setting $z = a + bi$. [3 marks]

**(b)** Find $z/\overline{z}$ in Cartesian form. [3 marks]

**5.** A monic real quadratic $z^2 + pz + q = 0$ has a root $2i$. [5 marks]

**(a)** Find $p$ and $q$. [3 marks]

**(b)** For a nonzero real $b$, state the monic real quadratic with roots $\pm bi$. [2 marks]

**6.** Let $p(z) = z^2 + az + b$ with real $a, b$, and suppose $p(1 + 2i) = 0$. [6 marks]

**(a)** Use substitution and component equations to find $a, b$. [4 marks]

**(b)** Verify that the conjugate is also a root. [2 marks]

**7.** A quadratic $q(z)$ has roots $-1 \pm i$ and satisfies $q(0) = 6$. [6 marks]

**(a)** Find its leading coefficient and expanded expression. [4 marks]

**(b)** Explain why the roots alone did not determine the leading coefficient. [2 marks]

**8.** Point $Z$ represents $z = a + 2i$, with real $a$. It is equally far from $1$ and $-3 + 4i$. [6 marks]

**(a)** Form an equation in $a$ using squared distances. [3 marks]

**(b)** Find $z$ and the common distance. [3 marks]

**9.** Take $z = a + bi$ and $w = c + di$, with real components. [6 marks]

**(a)** Express the real and imaginary components of $zw$. [2 marks]

**(b)** Prove $|zw| = |z||w|$. [4 marks]

**10.** A complex number $z$ has real part 2 and is at distance $\sqrt{10}$ from the point 1. [6 marks]

**(a)** Find all possible values of $z$. [3 marks]

**(b)** Find the monic real quadratic whose roots are these two values. [3 marks]