Base case and induction hypothesis

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Check the base case for $1 + 3 + \cdots + (2n - 1) = n^2$, for $n \geq 1$. [2 marks]

Both sides equal 1.

Answer: At $n = 1$, $1 = 1^2$.

2. State the induction hypothesis for $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$. [2 marks]

Use k consistently in every position.

Do not assume the next case.

Answer: For arbitrary integer $k \geq 1$, assume $\sum_{r=1}^{k} r^2 = k(k+1)(2k+1)/6$.

3. Check the first case of $3^n - 1$ even for integers $n \geq 0$. [2 marks]

The smallest allowed integer is 0.

Zero is even.

Answer: At $n = 0$, $3^0 - 1 = 0 = 2(0)$.

4. For positive integers n, write the induction hypothesis for $n^3 - n$ divisible by six. [2 marks]

The integer condition on q expresses divisibility.

Answer: Assume $k^3 - k = 6q$ for some integer q and arbitrary integer $k \geq 1$.

5. A theorem is stated for n at least 4, but the proof starts at n=5. Which case is omitted? [2 marks]

The successor implication cannot run backward.

Answer: n=4.

6. Why is an induction hypothesis not an assumption that the whole theorem is true? [3 marks]

P(k) is assumed temporarily for arbitrary k.

The base case and successor implication together establish every required case.

Answer: It is the premise in one conditional implication.