Practice worksheet | Mathematical induction

Identities involving powers and integers

Name: _____ Date: _____

6 questions | 36 marks

1. Prove $1 + 2 + 2^2 + \cdots + 2^{n-1} = (2^n - 1)$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

2. Prove $1 + 3 + 3^2 + \cdots + 3^{n-1} = (3^n - 1)/2$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

3. Prove $\sum_{r=1}^{n} r2^{r-1} = (n-1)2^n + 1$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

4. Prove $\prod_{r=1}^{n} (r+1)/r = n+1$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

5. Prove $\prod_{r=1}^{n}(2r-1)/(2r+1) = 1/(2n+1)$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

6. Prove $\sum_{r=1}^{n}(2r-1)^2 = n(4n^2-1)/3$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]