Mathematical induction - Practice test 6

Name: _____ Date: _____

10 questions | 56 marks | 75 minutes

Proof structure, distinct summation identities, integer divisibility and criticism of arguments.

No calculator needed.

1. A proof establishes P(5) and P(k) implies P(k+1) for every integer k at least 5. [4 marks]

(a) State the exact range established by the proof. [2 marks]

(b) Explain whether P(4) follows, and what is needed to claim all positive integers. [2 marks]

2. Prove $n^5 - n$ is divisible by 5 for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

3. Prove $\sum_{r=1}^{n} 1/[(2r-1)(2r+1)] = n/(2n+1)$ for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

4. Prove $\sum_{r=1}^{n} (1+2+\cdots+r) = n(n+1)(n+2)/6$ for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

5. Prove $10^n - 3^n$ is divisible by 7 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

6. The first case has been checked for $7^n - (-1)^n$ is divisible by 8. Complete the remaining induction argument. [6 marks]

(a) State exactly what is assumed at an arbitrary kth case. [1 mark]

(b) Derive the successor case. Identify where the hypothesis is used. [4 marks]

(c) Explain why the checked base plus this implication covers all positive integers. [1 mark]

7. Let $W_n = \sum_{r=1}^{n} r^2 2^r$ for positive integers n. [6 marks]

(a) Check the base case and state the hypothesis for $W_n = (n^2 - 2n + 3)2^{n+1} - 6$. [2 marks]

(b) Prove the successor formula and hence calculate W4. [4 marks]

8. Consider $R_n = \sum_{r=1}^{n} r^2/(4r^2 - 1)$ for positive integers n. [6 marks]

(a) Verify the first case of $R_n = n(n+1)/[2(2n+1)]$, and state the induction hypothesis and new term. [2 marks]

(b) Establish the successor case and conclude. [4 marks]

9. A computer checked $\prod_{r=1}^{n}(2r-1)/(2r+1) = 1/(2n+1)$ for the first one hundred positive integers. Replace that numerical evidence with a proof. [6 marks]

(a) Explain the limitation of the computer check and verify a valid starting case. [2 marks]

(b) State the hypothesis and derive the next case with exact algebra. [4 marks]

10. A divisibility proof concludes $A(k+1) = d(q+1/2)$ with integer q. Explain the problem. [4 marks]

(a) Identify the issue or give the required counterexample, with reasoning. [2 marks]

(b) State the required form of the conclusion. [2 marks]