Cartesian/polar forms

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Convert $1 + i$ to polar form. [2 marks]

$r = \sqrt{1 + 1} = \sqrt{2}$ and the equal positive coordinates give $\theta = \pi/4$.

Answer: $\sqrt{2}\,\text{cis}(\pi/4)$.

2. Convert $-2 - 2i$ to polar form with $-\pi < \theta \leq \pi$. [2 marks]

The modulus is $\sqrt{8} = 2\sqrt{2}$. Quadrant III has principal angle $-3\pi/4$.

Answer: $2\sqrt{2}\,\text{cis}(-3\pi/4)$.

3. Convert $6\text{cis}(\pi/6)$ to Cartesian form. [2 marks]

Multiply 6 by $\cos(\pi/6) = \sqrt{3}/2$ and $\sin(\pi/6) = 1/2$.

Answer: $3\sqrt{3} + 3i$.

4. A signal is represented by $5\text{cis}\,\pi$. State its real and imaginary components. [2 marks]

$5\cos\pi = -5$ and $5\sin\pi = 0$.

Answer: Real component -5; imaginary component 0.

5. A student writes $-1 + i = \sqrt{2}\,\text{cis}(-\pi/4)$. Explain and correct the error. [3 marks]

The given angle produces a positive real part and a negative imaginary part, opposite to those required.

Answer: The angle is in the wrong quadrant; use $\sqrt{2}\,\text{cis}(3\pi/4)$.

6. Why is an argument of $z = 0$ undefined? [2 marks]

Every value of θ gives $0\text{cis}\,\theta = 0$, so no direction is distinguished.

Answer: Zero has no direction from the origin.