De Moivre's theorem

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. Calculate $(\sqrt{3}+i)^3$. [2 marks]

$\sqrt{3}+i = 2\text{cis}(\pi/6)$, so its cube is $8\text{cis}(\pi/2) = 8i$.

Answer: $8i$.

2. Calculate $(1-i)^6$. [2 marks]

$1-i = \sqrt{2}\,\text{cis}(-\pi/4)$. The sixth power is $8\text{cis}(-3\pi/2) = 8i$.

Answer: $8i$.

3. Write $(2\text{cis}(\pi/3))^{-2}$ in polar form. [2 marks]

Raise the nonzero modulus to -2 and multiply the angle by -2.

Answer: $\frac{1}{4}\text{cis}(-2\pi/3)$.

4. Use de Moivre and a square expansion to obtain a formula for $\cos 2\theta$. [3 marks]

The real part of $(\cos\theta + i\sin\theta)^2$ is $\cos^2\theta - \sin^2\theta$; de Moivre gives real part $\cos 2\theta$.

Answer: $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

5. Show that $(\text{cis}(\pi/5))^{10} = 1$. [2 marks]

De Moivre gives $\text{cis}(10\pi/5) = \text{cis}(2\pi) = 1$.

Answer: The tenth power is 1.

6. Find the least positive integer n for which $(\text{cis}(2\pi/7))^n = 1$. [3 marks]

We require $2n\pi/7 = 2k\pi$, so $n = 7k$. The least positive integer is 7.

Answer: $n = 7$.