Powers and roots

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $z^2 = -9$. [2 marks]

Use arguments $(\pi + 2k\pi)/2$ for $k = 0, 1$ and modulus 3.

Answer: $z = 3i$ or $z = -3i$.

2. List the cube roots of 8 in polar form. [2 marks]

The angles are $2k\pi/3$ for $k = 0, 1, 2$.

Answer: $2,\ 2\text{cis}(2\pi/3),\ 2\text{cis}(4\pi/3)$.

3. Find all solutions of $z^4 = -16$. [3 marks]

The modulus is 2. Arguments come from $(\pi + 2k\pi)/4$.

Answer: $2\text{cis}(\pi/4 + k\pi/2)$, $k = 0, 1, 2, 3$.

4. Write the roots of $z^3 = 8i$ in principal-argument polar form. [3 marks]

Angles from $\pi/6 + 2k\pi/3$ are $\pi/6, 5\pi/6, 3\pi/2$; reduce the last by 2π.

Answer: $2\text{cis}(\pi/6),\ 2\text{cis}(5\pi/6),\ 2\text{cis}(-\pi/2)$.

5. Explain why $k = 4$ gives no new root in $z_k = 2\text{cis}(\pi/8 + k\pi/2)$. [2 marks]

The angle for $k = 4$ is $\pi/8 + 2\pi$, the same direction as $\pi/8$.

Answer: It repeats the $k = 0$ root.

6. The three cube roots of a nonzero number form an equilateral triangle. Explain using arguments. [3 marks]

Equal central angles on one circle give equal chord lengths, so all three sides are equal.

Answer: Adjacent roots differ in angle by $2\pi/3$ and have equal modulus.