Factorisation over the complex numbers

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Factorise $z^2 - 2z + 5$ over $\mathbb{C}$. [2 marks]

Complete the square: $(z-1)^2 + 4$, with roots $1 \pm 2i$.

Answer: $(z - (1 + 2i))(z - (1 - 2i))$.

2. Factorise $2z^2 + 8$ over $\mathbb{C}$. [2 marks]

Take out 2, then use the roots of $z^2 + 4 = 0$.

Answer: $2(z - 2i)(z + 2i)$.

3. Factorise $z^4 - 1$ completely over $\mathbb{C}$. [2 marks]

$z^4 - 1 = (z^2 - 1)(z^2 + 1)$; split both quadratics.

Answer: $(z - 1)(z + 1)(z - i)(z + i)$.

4. Solve $z^4 + 13z^2 + 36 = 0$. [3 marks]

Treat as a quadratic in z^2: $(z^2 + 4)(z^2 + 9) = 0$.

Answer: $z = \pm 2i, \ \pm 3i$.

5. Give the roots and multiplicities of $(z^2 + 1)^2(z - 2)$. [3 marks]

$z^2 + 1 = (z - i)(z + i)$; the outer square doubles both multiplicities.

Answer: i and $-i$ each have multiplicity 2; 2 has multiplicity 1.

6. Why do $z^2 + 1$ and $z^2 - 1$ have different real factorisations but both split over $\mathbb{C}$? [3 marks]

The available linear factors depend on which coefficient field is allowed.

Answer: The first has non-real roots $\pm i$; the second has real roots ± 1. All four values are complex numbers.