Conjugate-root results for real polynomials

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Find a real monic quadratic with roots $3 \pm i$. [2 marks]

$(z-3)^2 + 1 = z^2 - 6z + 10$.

Answer: $z^2 - 6z + 10$.

2. A monic real quartic has roots i, $2 + i$ and their conjugates. Factorise it over $\mathbb{R}$. [2 marks]

The conjugate pairs are $\pm i$ and $2 \pm i$.

Answer: $(z^2 + 1)(z^2 - 4z + 5)$.

3. A monic real cubic has root $1 + i$ and constant term -6. Find the remaining real root. [3 marks]

The conjugate pair gives $z^2 - 2z + 2$. Write $(z - r)(z^2 - 2z + 2)$; its constant term is $-2r = -6$.

Answer: 3.

4. Explain why a real polynomial of degree 5 cannot have exactly one non-real root counting multiplicity. [3 marks]

One non-real root would leave its forced conjugate uncounted.

Answer: Every non-real root must be accompanied by its distinct conjugate, so the non-real count is even.

5. For real a and b, $z^2 + az + b$ has root $1 - \sqrt{2}i$. Find a and b. [2 marks]

The pair gives $(z-1)^2 + 2 = z^2 - 2z + 3$.

Answer: $a = -2$, $b = 3$.

6. Test the claim: if i is a root of $P(z) = z - i$, then -i is also a root. [3 marks]

The coefficient $-i$ is non-real, so the conjugate-root theorem does not apply.

Answer: The claim is false: $P(-i) = -2i \neq 0$.