Complex Numbers - Paper 1

Name: _____ Date: _____

10 questions | 51 marks | 70 minutes

A balanced first paper on exact forms, arithmetic, powers, roots and simple geometry.

A scientific calculator may be used.

1. Let $z = -3 + 3\sqrt{3}i$. [4 marks]

(a) Find the modulus and principal argument. [2 marks]

(b) Write all arguments of z. [2 marks]

2. Let $u = 2 + i$ and $v = 1 - 3i$. [5 marks]

(a) Calculate uv in Cartesian form. [2 marks]

(b) Calculate u/v in Cartesian form. [3 marks]

3. Consider $a = 4\text{cis}(\pi/6)$ and $b = 2\text{cis}(\pi/3)$. [5 marks]

(a) Find ab in Cartesian form. [2 marks]

(b) Find a+b in Cartesian form. [3 marks]

4. A navigation diagram represents a displacement by $z = 3 - 4i$. A turn multiplies the displacement by i. [5 marks]

(a) Find the new displacement. [2 marks]

(b) Describe the turn and compare the two displacement lengths. [3 marks]

5. Use exact polar arithmetic. [5 marks]

(a) Calculate $(1 - \sqrt{3}i)^4$. [3 marks]

(b) Calculate $(2\text{cis}(\pi/4))^{-2}$. [2 marks]

6. Solve $z^3 = 27$. [6 marks]

(a) List all roots in polar form. [3 marks]

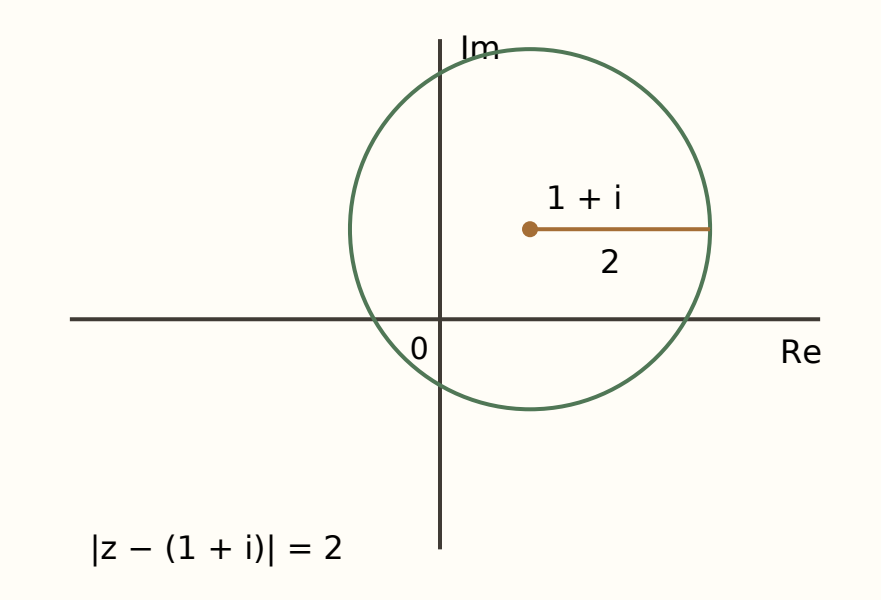
(b) Convert the two non-real roots to Cartesian form. [3 marks]

7. Let $\omega = \text{cis}(2\pi/3)$. [5 marks]

(a) Simplify $\omega^7 + \omega^5$. [2 marks]

(b) Find $|1 - \omega|$. [3 marks]

8. A locus is defined by $|z-(1+i)|=2$. [5 marks]



(a) State its centre and Cartesian equation. [2 marks]

(b) Find the points on this locus with real part 1. [3 marks]

9. Consider $P(z) = z^4 + 10z^2 + 9$. [5 marks]

(a) Factorise P into real quadratic factors. [2 marks]

(b) Factorise P completely over the complex numbers. [3 marks]

10. A monic polynomial of degree 3 has real coefficients and roots $2 + i$ and -1. [6 marks]

(a) State the third root, with a reason. [2 marks]

(b) Find the polynomial in expanded form. [4 marks]