Complex Numbers - Paper 2

Name: _____ Date: _____

10 questions | 55 marks | 70 minutes

A second balanced paper with signals, conjugates, roots, loci and reconstruction of polynomials.

A scientific calculator may be used.

1. Consider $z = 5\text{cis}(7\pi/6)$. [4 marks]

(a) Write z in Cartesian form. [2 marks]

(b) Give its principal argument and the principal argument of its conjugate. [2 marks]

2. Let $z = 1 - 2i$. [6 marks]

(a) Calculate z^2 and $z\bar{z}$. [3 marks]

(b) Find $1/z$ and verify by multiplication. [3 marks]

3. Two idealised same-frequency signals are represented by $A = 4\text{cis}(\pi/3)$ and $B = 4\text{cis}(-\pi/3)$. Their resultant is A+B. [5 marks]

(a) Find the resultant in Cartesian and polar forms. [3 marks]

(b) Explain why the resultant amplitude is not 8. [2 marks]

4. The map $w = (1 - i)z$ acts on the Argand plane. [5 marks]

(a) Describe its scale factor and rotation. [2 marks]

(b) Find the original point if its image is $w = 4i$. [3 marks]

5. Use de Moivre's theorem. [6 marks]

(a) Find $(\sqrt{3} + i)^6$. [3 marks]

(b) Find the least positive integer n for which $(\text{cis}(3\pi/5))^n = 1$. [3 marks]

6. Solve $z^4 = 81$. [6 marks]

(a) List every root in Cartesian form. [3 marks]

(b) Find the area of the square formed by these points. [3 marks]

7. Let $\zeta = \text{cis}(2\pi/5)$. [5 marks]

(a) Find ζ^{17} as a power from 0 to 4, and find $\overline{\zeta}$ as a power of ζ. [3 marks]

(b) Find $1 + \zeta + \zeta^2 + \zeta^3$. [2 marks]

8. Consider the locus $|z - 2| = |z + 2i|$. [6 marks]

(a) Find its Cartesian equation. [3 marks]

(b) Find its intersections with $|z| = \sqrt{2}$. [3 marks]

9. Let $P(z) = z^3 + z^2 + 4z + 4$. [6 marks]

(a) Show that -1 is a root and factorise over the real numbers. [3 marks]

(b) State all roots and factorise completely over the complex numbers. [3 marks]

10. A monic real quartic has roots $1 + i$ and $-1 + 2i$. [6 marks]

(a) Give its factorisation into real quadratic factors. [3 marks]

(b) Find its constant coefficient and the sum of all its roots without fully expanding. [3 marks]