Composition and domain restrictions

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Let $f(x) = \sqrt{x-1}$ and $g(x) = 2x + 3$. Find both compositions and their domains. [4 marks]

For f after g, require $2x + 3 \geq 1$.

For g after f, the inner root requires $x \geq 1$; g accepts every real output.

Answer: $f(g(x)) = \sqrt{2x+2}$, $x \geq -1$; $g(f(x)) = 2\sqrt{x-1} + 3$, $x \geq 1$.

2. Let $f(x) = 1/(x-2)$ and $g(x) = x^2$. Find the domain of $f \circ g$. [3 marks]

$f(g(x)) = 1/(x^2 - 2)$.

Require $x^2 - 2 \neq 0$.

Answer: All real x except $\pm\sqrt{2}$.

3. For $f(x) = \ln x$ and $g(x) = 3 - x$, find $(f \circ g)(x)$ and its domain. [3 marks]

The logarithm requires $3 - x > 0$.

Answer: $\ln(3 - x)$, $x < 3$.

4. Let $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$. Find the domain of $g \circ f$. [3 marks]

The inner square root must be defined.

After substitution, $g(f(x)) = x - 4$, but the original inner-domain restriction remains.

Answer: $x \geq 0$.

5. Let $f(x) = x + 1$ and $g(x) = 1/x$. Compare $f \circ g$ and $g \circ f$. [4 marks]

Substitute into the outer rule in each order.

The expressions and excluded inputs differ.

Answer: $1/x + 1$ with $x \neq 0$; $1/(x + 1)$ with $x \neq -1$.

6. Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = x - 1$. Find the domain of $f \circ g$. [3 marks]

Require $-2 \leq x - 1 \leq 2$.

Add 1 throughout.

Answer: $-1 \leq x \leq 3$.