One-to-one functions

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Is $f(x) = x^2$ one-to-one on the real numbers? Justify your answer. [2 marks]

Use $f(2) = f(-2) = 4$ even though the inputs differ.

Answer: No.

2. Show $f(x) = 3x - 7$ is one-to-one on the real numbers. [3 marks]

If $3a - 7 = 3b - 7$, then $3a = 3b$ and $a = b$.

Answer: It is one-to-one.

3. Is $f(x) = 1/x$, $x \neq 0$, one-to-one? [3 marks]

If $1/a = 1/b$ for nonzero a,b, multiplying by ab gives a=b.

Its separated branches do not share any y-value.

Answer: Yes.

4. Find two distinct inputs with the same output for $f(x) = x^3 - 3x$. [3 marks]

$x^3 - 3x = x(x^2 - 3)$.

Use two different roots.

Answer: 0 and $\sqrt{3}$ both give 0.

5. Give two maximal intervals on which $f(x) = (x - 2)^2$ is one-to-one. [3 marks]

The graph is strictly decreasing on the first interval and increasing on the second.

Each restriction passes the horizontal-line test.

Answer: $(-\infty, 2]$ and $[2, \infty)$.

6. Explain why $f(x) = 2^x$ has at most one input for each output. [2 marks]

For a<b, raising a base greater than 1 to these powers gives $2^a < 2^b$.

A horizontal line meets its graph at most once.

Answer: It is strictly increasing.