Absolute-value graphs

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Write $f(x) = |x - 2| - 3$ piecewise and find its intercepts. [4 marks]

Split at x=2.

Solve each valid linear branch for y=0.

Answer: $f(x) = -x - 1$ for $x < 2$, and $x - 5$ for $x \geq 2$; roots $-1, 5$, y-intercept -1.

2. Find the turning points and zeros of $y = |x^2 - 4|$. [4 marks]

For $|x| < 2$, the rule is $4 - x^2$.

Outside that interval the rule is $x^2 - 4$.

Answer: Zeros $x = \pm 2$; corner minima $(\pm 2, 0)$; local maximum $(0, 4)$.

3. Solve $|2x - 1| = 5$. [3 marks]

$2x - 1 = 5$ or $2x - 1 = -5$.

Check both values in the original equation.

Answer: $x = 3$ or $x = -2$.

4. For $f(x) = x - 1$, compare $|f(x)|$ and $f(|x|)$. [3 marks]

The first reflects negative output upward.

The second mirrors the original right-hand graph into the left half-plane.

Answer: $|x - 1|$ has vertex $(1, 0)$; $|x| - 1$ has vertex $(0, -1)$.

5. Give the domain, range and symmetry of $y = |1/x|$. [3 marks]

It equals $1/|x|$.

It is positive and unchanged when x is replaced by -x.

Answer: Domain $x \neq 0$; range $y > 0$; even symmetry.

6. Solve $|x + 2| < 3$ and explain the interval. [3 marks]

$-3 < x + 2 < 3$.

Subtract 2; the distance from -2 must be less than 3.

Answer: $-5 < x < 1$.