Rational functions

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Find the hole in $f(x) = (x^2 - 1)/(x - 1)$. [3 marks]

For x not equal to 1, simplify to x+1.

The excluded original input remains excluded.

Answer: $(1, 2)$.

2. Divide $(x^2 + 1)/(x - 1)$ and state the oblique asymptote. [3 marks]

Polynomial division gives quotient x+1 and remainder 2.

The remainder term tends to zero as the magnitude of x grows.

Answer: $x + 1 + 2/(x - 1)$; asymptote $y = x + 1$.

3. Analyse $f(x) = (2x + 1)/(x - 3)$: give asymptotes and intercepts. [4 marks]

Rewrite as $2 + 7/(x - 3)$.

Set the numerator to zero for the x-intercept; substitute x=0 for the other.

Answer: Asymptotes $x = 3$, $y = 2$; intercepts $(-1/2, 0)$ and $(0, -1/3)$.

4. Find the stationary points and range of $f(x) = x/(x^2 + 1)$. [4 marks]

$f'(x) = (1 - x^2)/(x^2 + 1)^2$.

The sign changes at -1 and 1 establish the extrema.

Answer: Minimum $(-1, -1/2)$, maximum $(1, 1/2)$; range $[-1/2, 1/2]$.

5. For $f(x) = (x - 2)/(x^2 - 4)$, state the domain, simplified rule and discontinuities. [4 marks]

Factor the denominator into (x-2)(x+2).

Cancellation does not restore the excluded input 2.

Answer: Domain excludes ± 2; $f(x) = 1/(x + 2)$ for allowed x; pole at -2 and hole at $(2, 1/4)$.

6. Can a graph cross a horizontal asymptote? Use $f(x) = x/(x^2 + 1)$ to justify your answer. [2 marks]

A horizontal asymptote describes end behaviour, not a forbidden y-value.

Answer: Yes; it crosses its horizontal asymptote y=0 at the origin.