Intercepts, stationary behaviour and asymptotes

6 questions | 26 marks

Equivalent correct methods earn the corresponding marks.

1. For $f(x) = x^2/(x-1)$, find asymptotes and stationary points. [5 marks]

Division gives $x + 1 + 1/(x-1)$.

$f'(x) = x(x-2)/(x-1)^2$.

The derivative changes from positive to negative at 0 and negative to positive at 2.

Answer: Asymptotes $x = 1$, $y = x + 1$; local maximum $(0, 0)$, local minimum $(2, 4)$.

2. For $f(x) = (x^2 - 4)/(x^2 - 1)$, give intercepts, asymptotes and its stationary point. [5 marks]

Rewrite as $1 - 3/(x^2 - 1)$.

$f'(x) = 6x/(x^2 - 1)^2$, changing from negative to positive at 0.

Answer: Roots ± 2; y-intercept 4; vertical asymptotes ± 1; horizontal asymptote 1; local minimum $(0, 4)$.

3. Find the stationary points of $f(x) = x + 1/x$ and classify them. [4 marks]

$f'(x) = 1 - 1/x^2$.

The critical inputs are -1 and 1, with x=0 outside the domain.

Use derivative signs on the separate branches.

Answer: Local maximum $(-1, -2)$ and local minimum $(1, 2)$.

4. For $f(x) = x^3/(x^2 - 1)$, explain why the origin is not a local extremum. [4 marks]

$f'(x) = x^2(x^2 - 3)/(x^2 - 1)^2$.

Its sign does not change at 0; the odd graph has a stationary inflection there.

Answer: The derivative vanishes at 0 but is negative immediately on both sides.

5. Find all x-intercepts of $f(x) = (x + 1)/(x^2 - 1)$ and explain any excluded apparent intercept. [3 marks]

The numerator vanishes only at x=-1, which is excluded by the denominator.

The simplified reciprocal rule has no zero.

Answer: There are no x-intercepts.

6. Find the range of $f(x) = x^2/(x - 1)$ using its extrema or a discriminant. [5 marks]

$y(x - 1) = x^2$ gives $x^2 - yx + y = 0$.

Real x requires $y^2 - 4y \geq 0$, so y is at most 0 or at least 4.

The excluded input x=1 is never a root of this equation.

Answer: $(-\infty, 0] \cup [4, \infty)$.