Combining graphical/algebraic/calculus information in sketches

6 questions | 23 marks

Equivalent correct methods earn the corresponding marks.

1. Sketch $y = x + 1/x$ using its domain, extrema and asymptotes. [5 marks]

The equation is undefined at 0.

The difference from y=x is 1/x, tending to zero at either end.

Derivative signs determine each branch shape.

Answer: Two odd-symmetric branches; x=0 is vertical, y=x is oblique; extrema $(-1, -2)$ and $(1, 2)$; no intercepts.

2. Give a complete sketch description of $y = |x - 2| - 3$. [4 marks]

Use the two linear branches and place the exact intercepts.

The function is continuous but not differentiable at x=2.

Answer: V shape with vertex $(2, -3)$, roots -1 and 5, y-intercept -1; slopes -1 then 1; range $[-3, \infty)$.

3. A student draws one continuous curve through x=1 for $y = x^2/(x - 1)$. Explain and correct the sketch. [3 marks]

The original denominator is zero at 1.

The numerator is nonzero there, giving a pole rather than a hole.

Answer: The graph must be broken into branches separated by the vertical asymptote x=1.

4. For $y = 1/(x^2 + 1)$, justify the complete graph from symmetry, derivative and limits. [4 marks]

$y' = -2x/(x^2 + 1)^2$.

The derivative is positive for negative x and negative for positive x.

The denominator never vanishes.

Answer: An even positive curve, maximum $(0, 1)$, approaching y=0 at both ends with no vertical asymptote.

5. Two formulas simplify to the same expression. Explain why their graphs might still differ. [3 marks]

Compare $x + 1$ with $(x^2 - 1)/(x - 1)$.

Only the second excludes x=1.

Answer: Their original domains may differ, producing a hole in one graph.

6. Find the tangent to $f(x) = x/(x^2 + 1)$ at x=0 and explain why it does not determine the whole graph. [4 marks]

$f(0) = 0$ and $f'(0) = 1$.

A tangent is local information; extrema and end behaviour need separate analysis.

Answer: $y = x$.