Functions and graph sketching - Practice test 1

Name: _____ Date: _____

10 questions | 46 marks | 70 minutes

Ten multipart questions combining exact calculations, domains, graph evidence, interpretation and reasoning. Show working and label key graph features.

A scientific calculator may be used.

1. Let $f(x) = \sqrt{x+2}$ and $g(x) = 3x - 1$. [4 marks]

(a) Find $(f \circ g)(x)$ and its domain. [2 marks]

(b) Find $(g \circ f)(x)$ and its domain. Are the two functions equal? [2 marks]

2. Consider $f(x) = x^2 - 4x + 1$ on the real numbers. [4 marks]

(a) Use two distinct inputs to show that f is not one-to-one. [2 marks]

(b) Give the largest interval starting at the vertex on which f is increasing, and state its range. [2 marks]

3. Let $f(x) = 5 - 2x$ on the real numbers. [4 marks]

(a) Find f^{-1} and verify one of the inverse compositions. [2 marks]

(b) Find the point where the graph of f meets the graph of its inverse. [2 marks]

4. Let $h(x) = |2x + 2| - 4$. [5 marks]

(a) Write h piecewise and find its vertex. [2 marks]

(b) Find the intercepts and solve $h(x) < 0$. [3 marks]

5. Let $r(x) = 1/(x-2)^2$. [5 marks]

(a) State its domain, asymptotes and range. [3 marks]

(b) State its symmetry and solve $r(x) = 1$. [2 marks]

6. Consider $q(x) = (x^2 - 9)/(x - 3)$. [4 marks]

(a) Simplify the rule while retaining the original domain. [2 marks]

(b) Give the hole and both intercepts. [2 marks]

7. For $f(x) = x + 4/x$, with $x \neq 0$: [6 marks]

(a) Find and classify the stationary points. [3 marks]

(b) Give the asymptotes and justify the complete range. [3 marks]

8. Let $u(x) = 1/(x^2 + 4)$. [5 marks]

(a) Use differentiation and symmetry to locate its maximum. [2 marks]

(b) Describe a complete sketch, including intercepts and end behaviour. [3 marks]

9. A sensor uses output $f(t) = \sqrt{t-2}$ for time $t \geq 2$. A clock display reports $t = 3s + 2$. [5 marks]

(a) Express the output in s and give its domain. [3 marks]

(b) Find the displayed s when output is 6. [2 marks]

10. A student states that $(x^2 - 1)/(x - 1)$ and x+1 are the same function on all real inputs. [4 marks]

(a) Identify the false part of the claim. [2 marks]

(b) Describe the graphical difference. [2 marks]