Functions and graph sketching - Practice test 3

Name: _____ Date: _____

10 questions | 49 marks | 70 minutes

Ten multipart questions combining exact calculations, domains, graph evidence, interpretation and reasoning. Show working and label key graph features.

A scientific calculator may be used.

1. Let $f(x) = \ln x$ and $g(x) = x^2 - 4$. [4 marks]

(a) Find the rule and domain of $f \circ g$. [2 marks]

(b) Find the rule and domain of $g \circ f$ and compare its symmetry with the first composition. [2 marks]

2. Let $f(x) = x^3 - 3x$. [5 marks]

(a) Find its intervals of increase and decrease. [3 marks]

(b) How many solutions does $f(x) = 1$ have? Justify without finding the roots. [2 marks]

3. Let $f(x) = 2e^{x-1} + 3$. [5 marks]

(a) Find its inverse, domain and range. [3 marks]

(b) Find the exact input for output 7. [2 marks]

4. Consider $y = |x - 1| + |x + 1|$. [5 marks]

(a) Write a piecewise rule on the three intervals separated by -1 and 1. [3 marks]

(b) Find the range and all solutions of $y = 2$ and $y = 4$. [2 marks]

5. Let $r(x) = 1/(x^2 - 4)$. [5 marks]

(a) Find the asymptotes and classify its stationary point. [3 marks]

(b) Find all points fixed by replacing the ordinate $r(x)$ with its reciprocal. [2 marks]

6. Let $f(x) = (x^2 - 1)/(x^2 + x)$. [5 marks]

(a) Simplify and locate the hole and pole. [3 marks]

(b) Find both intercepts and the full range. [2 marks]

7. For $f(x) = x/(x^2 + 4)$: [5 marks]

(a) Find and classify all stationary points. [3 marks]

(b) Find every solution of $f(x) = \frac{1}{5}$. [2 marks]

8. Let $g(x) = x^3/(x^2 + 1)$. [5 marks]

(a) Find its asymptote and classify the stationary point at zero. [3 marks]

(b) Explain why the graph is one-to-one even though $g'(0) = 0$. [2 marks]

9. A charge model is $C(x) = x + 4/x$ dollars, with batch size $x > 0$. [6 marks]

(a) Find the stationary point. [3 marks]

(b) Show it is a minimum and interpret it. [3 marks]

10. For $f(x) = x - 3$, a student says $f(|x|)$ is never negative. [4 marks]

(a) Give a counterexample. [2 marks]

(b) Explain the confusion. [2 marks]