Closest points and distances including skew lines

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. Find the closest point on (t,0,0) to (3,4,5) and its distance. [2 marks]

Perpendicularity requires t=3.

The remaining displacement is (0,4,5).

Answer: Q=(3,0,0), distance $\sqrt{41}$.

2. Find the distance from (1,2,3) to x+y+z=0. [2 marks]

Use |6|/$\sqrt{3}$.

Simplify to $2\sqrt{3}$.

Answer: $2\sqrt{3}$

3. Find the foot of the perpendicular from (1,2,3) to x+y+z=0. [2 marks]

Subtract [6/3]$(1, 1, 1)$ from the point.

The result lies in the plane.

Answer: $(-1, 0, 1)$

4. Find the closest points and distance between $\ell_1 = s(1, 1, 0)$ and $\ell_2 = (0, 2, 1) + t(1, 0, 1)$. [4 marks]

The joining vector is $(t - s, 2 - s, 1 + t)$. Perpendicularity gives $t + 2 - 2s = 0$ and $2t - s + 1 = 0$.

Solve to obtain s=1,t=0. The joining vector is (-1,1,1), which has length $\sqrt{3}$.

Answer: $P = (1, 1, 0)$ and $Q = (0, 2, 1)$; distance $\sqrt{3}$.

5. Find the distance between (s,0,0) and (t,3,4). [2 marks]

The lines are parallel to the x axis.

The perpendicular displacement is (0,3,4).

Answer: 5

6. For line (1,0,0)+t(1,1,0) and P(0,1,2), find the closest t and distance. [2 marks]

The dot product $(1 + t, t - 1, -2) \cdot (1, 1, 0) = 2t$ vanishes at 0.

Then the squared distance is $1 + 1 + 4 = 6$.

Answer: $t = 0$, distance $\sqrt{6}$.