Inverse-trigonometric derivatives and integrals

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\int 3/(9+x^2)\,dx$. [2 marks]

Use the arctangent rule with a=3.

The factor 3 cancels the outside 1/3.

Answer: $\arctan(x/3)+C$.

2. Find $\int 2/\sqrt{1-4x^2}\,dx$. [2 marks]

Use u=2x and du=2dx.

The transformed integral is $\int (1-u^2)^{-1/2}\,du$.

Answer: $\arcsin(2x)+C$, $|x|<1/2$.

3. Find $\int 1/(1+9x^2)\,dx$. [2 marks]

Differentiate arctan(3x) to see the extra factor 3.

Divide by 3.

Answer: $\frac{1}{3}\arctan(3x)+C$.

4. Evaluate $\int_0^{1/2} 1/\sqrt{1-x^2}\,dx$. [2 marks]

The antiderivative is arcsin x.

Evaluate arcsin(1/2)-arcsin(0).

Answer: $\pi/6$.

5. Find $\int 1/(x^2+16)\,dx$. [2 marks]

Use a=4 in the arctangent pattern.

Check the chain factor.

Answer: $\frac{1}{4}\arctan(x/4)+C$.

6. Differentiate arccos(x/2) on -2<x<2. [2 marks]

Use derivative $-u'/\sqrt{1-u^2}$.

Simplify with u=x/2.

Answer: $-1/\sqrt{4-x^2}$.