Trigonometric identities in integration

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\int \cos^2(2x)\,dx$. [2 marks]

Use $(1+\cos 4x)/2$.

Divide the cosine antiderivative by its frequency 4.

Answer: $x/2+\sin 4x/8+C$.

2. Find $\int \sin x \cos x\,dx$. [2 marks]

Use u=sin x.

Integrate u du.

Answer: $\sin^2 x/2+C$.

3. Find $\int \sin^2 x \cos^2 x\,dx$. [2 marks]

Rewrite as $(1-\cos 4x)/8$.

Integrate each term.

Answer: $x/8-\sin 4x/32+C$.

4. Evaluate $\int_0^{\pi/2} \sin^2 x\,dx$. [2 marks]

Use $x/2 - \sin 2x/4$.

The sine endpoint terms are zero.

Answer: $\pi/4$.

5. Find $\int \sin x \cos(3x)\,dx$. [2 marks]

Use $\sin x \cos 3x = (\sin 4x - \sin 2x)/2$.

Integrate both terms with their frequencies.

Answer: $-\cos 4x/8 + \cos 2x/4 + C$.

6. Find $\int \cos^3 x\,dx$. [2 marks]

Write $(1 - \sin^2 x)\cos x$.

Use u=sin x.

Answer: $\sin x - \sin^3 x/3 + C$.