Simple partial fractions

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Integrate $1/[x(x+1)]$. [2 marks]

Use the difference of reciprocal terms.

Integrate each and exclude x=0,-1.

Answer: $\ln|x| - \ln|x+1| + C$.

2. Integrate $2/(x^2-4)$. [2 marks]

Decompose as $1/[2(x-2)] - 1/[2(x+2)]$.

Integrate away from the poles.

Answer: $\frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x+2| + C$.

3. Integrate $(5x+1)/[(x-1)(x+1)]$. [2 marks]

Use coefficients A=3,B=2.

Their cleared numerator is 3(x+1)+2(x-1).

Answer: $3\ln|x-1| + 2\ln|x+1| + C$.

4. Integrate $(2x+3)/(x+1)^2$. [2 marks]

Write $2/(x+1)+1/(x+1)^2$.

Integrate by logarithm and power rules.

Answer: $2\ln|x+1|-1/(x+1)+C$.

5. Integrate $(x^2+1)/(x^2-1)$. [2 marks]

Divide to obtain $1+2/(x^2-1)$.

Decompose the remaining fraction.

Answer: $x+\ln|x-1|-\ln|x+1|+C$.

6. Evaluate $\int_1^2 1/[x(x+1)]\,dx$. [2 marks]

Use $\ln x-\ln(x+1)$ on this positive interval.

Endpoint subtraction gives $\ln(2/3)-\ln(1/2)$.

Answer: $\ln(4/3)$.