Selecting/combining methods

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Integrate $\ln x/x$ for x>0. [2 marks]

Set u=ln x.

Integrate u du.

Answer: $(\ln x)^2/2 + C$.

2. Integrate $x/\sqrt{1-x^2}$ on -1<x<1. [2 marks]

Use u=1-x squared.

Then x dx=-du/2.

Answer: $-\sqrt{1-x^2} + C$.

3. Integrate xe^{2x}. [2 marks]

Use parts with u=x and v=exp(2x)/2.

Subtract the remaining exponential integral.

Answer: $e^{2x}(x/2 - 1/4) + C$.

4. Integrate $\cos^2 x \sin x$. [2 marks]

Use u=cos x.

The negative differential changes the sign.

Answer: $-\cos^3 x/3 + C$.

5. Explain whether x exp(x squared) and x exp(x) should use the same first method. [2 marks]

The derivative of x squared supplies the first factor x.

Differentiating x simplifies the second product without such a match.

Answer: No. Substitution is efficient for the first; parts is efficient for the second.

6. Verify that $e^x(x^2 - 2x + 2)$ integrates $x^2 e^x$. [2 marks]

Differentiate by the product rule.

The bracket sum simplifies to x squared.

Answer: It does.