Volumes of revolution about either coordinate axis

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Rotate 0<=x<=1, 0<=y<=x squared about the x axis. [2 marks]

The radius squared is x to the fourth.

Integrate pi times x to the fourth from 0 to 1.

Answer: $\pi/5$ cubic units

2. Rotate 0<=x<=sqrt(2), x squared<=y<=2 about the x axis. [2 marks]

Use washer radii 2 and x squared.

Integrate $\pi(4 - x^4)$ to sqrt(2).

Answer: $16\sqrt{2}\pi/5$ cubic units

3. Rotate 0<=y<=3, 0<=x<=y about the y axis. [2 marks]

The radius is y.

$\pi\int_0^3 y^2\,dy = 9\pi$.

Answer: 9π cubic units

4. Rotate 1<=x<=2, 1<=y<=x about the x axis. [2 marks]

Use outer radius x and inner radius 1.

Integrate $\pi(x^2 - 1)$ from 1 to 2.

Answer: $4\pi/3$ cubic units

5. Rotate the rectangle 1<=x<=2, 0<=y<=3 about the y axis. [2 marks]

The annular cross-section area is $\pi(4 - 1) = 3\pi$.

Multiply by height 3.

Answer: 9π cubic units

6. The region under y=kx from x=0 to 1 is rotated about the x axis, with k>0. If volume is 12pi, find k. [2 marks]

The volume is $\pi k^2/3$.

Solve k squared=36 and use k>0.

Answer: $k = 6$.