Integration techniques and applications - practice test 1

10 questions | 44 marks | 75 minutes

Select methods and establish reliable exact working.

Equivalent correct methods earn the corresponding marks.

1. Let $F'(x) = 2x + \frac{3}{x^2}$ for x>0. [4 marks]

(a) Find the general antiderivative. [2 marks]

Use the power rule term by term.

Keep the reciprocal term on its positive-domain interval.

Answer: $\frac{x^3 - 3}{x} + C$.

(b) Find F when F(1)=5. [2 marks]

At x=1, the nonconstant terms total -2.

Thus $C = 7$.

Answer: $x^2 + 7 - \frac{3}{x}$.

2. Consider $f(x) = 1/(4 + x^2)$. [4 marks]

(a) Find an antiderivative. [2 marks]

Recognise the sum-of-squares denominator.

Use $a = 2$ in the arctangent formula.

Answer: $\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2} + C$.

(b) Find the exact integral from x=0 to x=2. [2 marks]

At the upper endpoint, arctan(1)=pi/4.

The lower endpoint contributes zero.

Answer: $\pi/8$.

3. Consider $\int x(1+x^2)^2\,dx$. [5 marks]

(a) Use a substitution to find the general antiderivative. [3 marks]

Set u=1+x squared and du=2x dx.

Integrate $u^2/2$ and return to x.

Answer: $\frac{(x^2+1)^3}{6}+C$.

(b) Evaluate the integral from 0 to 1 using transformed limits. [2 marks]

The new limits are u=1 and u=2.

Endpoint subtraction gives $\frac{7}{6}$.

Answer: $\frac{7}{6}$.

4. Consider $\int \sin^2(x)\,dx$. [4 marks]

(a) Rewrite the integrand and integrate. [2 marks]

Use $\sin^2(x)=(1-\cos(2x))/2$.

Account for the cosine frequency in the antiderivative.

Answer: $\frac{x}{2}-\frac{\sin(2x)}{4}+C$.

(b) Evaluate it from 0 to $\pi/2$. [2 marks]

The sine endpoint term is zero at both bounds.

The remaining term is half the interval length.

Answer: $\pi/4$.

5. For x>=0, integrate $(5x + 13)/[(x + 2)(x + 3)]$. [5 marks]

(a) Find the partial fractions. [2 marks]

Clear denominators and compare coefficients.

The coefficients are A=3, B=2.

Answer: $3/(x + 2) + 2/(x + 3)$.

(b) Find the general antiderivative, then its integral from 0 to 1. [3 marks]

The logarithm arguments are positive on the stated interval.

Subtract the endpoint values.

Answer: $3\ln(x + 2) + 2\ln(x + 3) + C$; definite value $\ln(6)$.

6. Find $\int xe^x\,dx$. [5 marks]

(a) Use integration by parts, stating u and dv. [3 marks]

Choose u=x and $dv = e^x dx$, so $v = e^x$.

Subtract the remaining exponential integral.

Answer: $(x - 1)e^x + C$.

(b) Evaluate the integral from 0 to 1. [2 marks]

Substitute both endpoints in the antiderivative.

Check the result is positive.

Answer: 1.

7. Consider $\int 1/(x^2+2x+5)\,dx$. [4 marks]

(a) Complete the square and find an antiderivative. [2 marks]

The denominator is $(x+1)^2+4$.

Apply the shifted arctangent pattern.

Answer: $\frac{\operatorname{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}+C$.

(b) Evaluate it from x=-1 to x=1. [2 marks]

The arctangent arguments run from 0 to 1.

Multiply pi/4 by the outside factor 1/2.

Answer: *pi*/8.

8. The curve is $y=x(x-2)$ on $0\le x\le 2$. [4 marks]

(a) Evaluate its signed integral over the interval. [2 marks]

The antiderivative is x cubed/3 minus 2x squared/2.

Subtract endpoints.

Answer: $-\frac{4}{3}$.

(b) Find the geometric area and explain the sign difference. [2 marks]

The curve lies below the x axis between its zeros.

Area is the negative of this single below-axis integral.

Answer: $\frac{4}{3}$ square units.

9. Consider $y = 2x$ and $y = x^2$ in the first quadrant. [4 marks]

(a) Find both intersections and identify the upper curve between them. [2 marks]

Solve x squared=2x.

A positive midpoint shows the line is above the parabola.

Answer: $(0, 0)$ and $(2, 4)$; the line is upper.

(b) Find the bounded area. [2 marks]

Integrate line minus parabola between the roots.

The result is $\frac{4}{3}$.

Answer: $\frac{4}{3}$ square units.

10. The triangular region $0 \le x \le 2$, $0 \le y \le x$ is rotated. [5 marks]

(a) Find its volume about the x axis. [2 marks]

The disk radius is x.

Use $\pi\int_0^2 x^2 dx$.

Answer: $\frac{8\pi}{3}$ cubic units.

(b) Find its volume about the y axis using horizontal washers. [3 marks]

At height y, the radii are 2 and y.

Use $\pi\int_0^2 (4 - y^2) dy$.

Answer: $\frac{16\pi}{3}$ cubic units.