Integration techniques and applications - practice test 1

Name: _____ Date: _____

10 questions | 44 marks | 75 minutes

Select methods and establish reliable exact working.

A scientific calculator may be used.

1. Let $F'(x) = 2x + \frac{3}{x^2}$ for x>0. [4 marks]

(a) Find the general antiderivative. [2 marks]

(b) Find F when F(1)=5. [2 marks]

2. Consider $f(x) = 1/(4 + x^2)$. [4 marks]

(a) Find an antiderivative. [2 marks]

(b) Find the exact integral from x=0 to x=2. [2 marks]

3. Consider $\int x(1+x^2)^2\,dx$. [5 marks]

(a) Use a substitution to find the general antiderivative. [3 marks]

(b) Evaluate the integral from 0 to 1 using transformed limits. [2 marks]

4. Consider $\int \sin^2(x)\,dx$. [4 marks]

(a) Rewrite the integrand and integrate. [2 marks]

(b) Evaluate it from 0 to $\pi/2$. [2 marks]

5. For x>=0, integrate $(5x + 13)/[(x + 2)(x + 3)]$. [5 marks]

(a) Find the partial fractions. [2 marks]

(b) Find the general antiderivative, then its integral from 0 to 1. [3 marks]

6. Find $\int xe^x\,dx$. [5 marks]

(a) Use integration by parts, stating u and dv. [3 marks]

(b) Evaluate the integral from 0 to 1. [2 marks]

7. Consider $\int 1/(x^2 + 2x + 5)\,dx$. [4 marks]

(a) Complete the square and find an antiderivative. [2 marks]

(b) Evaluate it from x=-1 to x=1. [2 marks]

8. The curve is $y = x(x - 2)$ on $0 \leq x \leq 2$. [4 marks]

(a) Evaluate its signed integral over the interval. [2 marks]

(b) Find the geometric area and explain the sign difference. [2 marks]

9. Consider $y = 2x$ and $y = x^2$ in the first quadrant. [4 marks]

(a) Find both intersections and identify the upper curve between them. [2 marks]

(b) Find the bounded area. [2 marks]

10. The triangular region $0 \leq x \leq 2$, $0 \leq y \leq x$ is rotated. [5 marks]

(a) Find its volume about the x axis. [2 marks]

(b) Find its volume about the y axis using horizontal washers. [3 marks]