Integration techniques and applications - practice test 2

10 questions | 46 marks | 75 minutes

Combine techniques and interpret geometric and practical results.

Equivalent correct methods earn the corresponding marks.

1. A model has $F'(x) = 4e^{2x}$. [4 marks]

(a) Find F if F(0)=5. [2 marks]

Integrate the exponential and divide by its inner coefficient.

F(0)=2+C=5, so C=3.

Answer: $F = 2e^{2x} + 3$.

(b) Evaluate $\int_0^{\frac{\ln(2)}{2}} 4e^{2x}\,dx$. [2 marks]

Use the antiderivative without its arbitrary constant.

The endpoint difference is 2.

Answer: 2.

2. For $|x| < 1/2$, consider $f(x) = 2/\sqrt{1-4x^2}$. [4 marks]

(a) Find an antiderivative and explain the inner factor. [2 marks]

The derivative of the inner argument is 2.

It matches the numerator.

Answer: $\arcsin(2x) + C$.

(b) Evaluate $\int_0^{1/4} f(x)\,dx$. [2 marks]

The inverse-sine argument at the upper endpoint is 1/2.

Use radians.

Answer: *pi*/6.

3. For x>0, consider $(2x+5)/(x+2)^2$. [4 marks]

(a) Decompose it using both powers of the repeated factor. [2 marks]

Write the numerator as $2(x+2)+1$.

Divide each term by the common squared denominator.

Answer: $2/(x+2)+1/(x+2)^2$.

(b) Integrate and verify by differentiation. [2 marks]

Use a logarithm for the reciprocal term and the power rule for the squared reciprocal.

Differentiation recovers the original expression.

Answer: $\frac{2(x+2)\ln(x+2)-1}{x+2}+C$.

4. For x>0, consider $\int x^1 \ln x\,dx$. [5 marks]

(a) Find the general antiderivative. [3 marks]

Choose u=ln x and integrate the power as dv.

Simplify the remaining power integral.

Answer: $\frac{x^2(2\ln(x)-1)}{4}+C$.

(b) Evaluate it from x=1 to x=e. [2 marks]

Use ln e=1 and ln 1=0.

Subtract the complete lower endpoint value.

Answer: $\frac{1}{4}+\frac{e^2}{4}$.

5. Consider $\int \sin^3(x)\,dx$. [5 marks]

(a) Find an antiderivative using an identity and substitution. [3 marks]

Keep one sine factor and replace the other square with $1-\cos^2(x)$.

Set $u=\cos(x)$ and include the factor -1.

Answer: $\frac{(\cos(2x)-5)\cos(x)}{6}+C$.

(b) Evaluate it from 0 to $\pi/2$. [2 marks]

The endpoint cosine values are 1 and 0.

Substitute in the antiderivative.

Answer: $\frac{2}{3}$.

6. Consider $\int x\ln(x^2+1)\,dx$. [6 marks]

(a) Use substitution followed by parts to integrate. [4 marks]

Set $u=x^2+1$ so $x\,dx=du/2$.

Integrate ln u by parts.

Answer: $-\frac{x^2}{2}+\frac{(x^2+1)\ln(x^2+1)}{2}-\frac{1}{2}+C$.

(b) Evaluate the original integral from 0 to 1. [2 marks]

Use transformed limits 1 and 2.

Subtract complete endpoint expressions.

Answer: $-\frac{1}{2}+\ln(2)$.

7. A region is bounded by $y=3x-x^2$ and the x axis. [5 marks]

(a) Find its horizontal endpoints. [2 marks]

Factor the quadratic as x(3-x).

Answer: $x=0$ and $x=3$.

(b) Find the exact region area. [3 marks]

The curve is above the axis between these roots.

Integrate it to obtain $\frac{9}{2}$.

Answer: $\frac{9}{2}$ square units.

8. A region is $0 \le y \le 2$ and $0 \le x \le y^2$. [5 marks]

(a) Find its plane area. [2 marks]

Use horizontal strips of width y squared.

Answer: $\frac{8}{3}$ square units.

(b) Find its volume about the y axis. [3 marks]

The disk radius is y squared, so the squared radius is y to the fourth.

Integrate πy^4 from 0 to 2.

Answer: $\frac{32\pi}{5}$ cubic units.

9. A tank starts with 20 L. Water enters at $q(t) = 2t + 2$ L/min for $0 \le t \le 3$. [4 marks]

(a) Find the added water and the final amount. [2 marks]

Integrate q over the three-minute interval.

Add the initial amount only after calculating the change.

Answer: Added 15 L; final 35 L.

(b) Find the average inflow rate. [2 marks]

Divide accumulated inflow by elapsed time.

The units remain litres per minute.

Answer: 5 L/min.

10. A student claims $G(x) = xe^{2x}$ is an antiderivative of e^{2x}. [4 marks]

(a) Differentiate G and identify the mistake. [2 marks]

Use the product rule.

The extra product term cannot be discarded.

Answer: $G' = e^{2x}(1 + 2x)$, so the claim is false.

(b) Give a correct antiderivative of xe^{2x}. [2 marks]

Apply integration by parts.

Verify the result by differentiation.

Answer: $e^{2x}(x/2 - 1/4) + C$.