Integration techniques and applications - practice test 4

Name: _____ Date: _____

10 questions | 54 marks | 75 minutes

Mixed techniques with translated inverse functions, repeated parts, horizontal regions and accumulated rates.

A scientific calculator may be used.

1. For all real x, let $f(x) = (x+1)/\sqrt{x^2+2x+5}$. [5 marks]

(a) Find an antiderivative using substitution. [3 marks]

(b) Find the member F for which F(0)=3. [2 marks]

2. Consider $f(x) = 1/\sqrt{4-(x-1)^2}$. [5 marks]

(a) State its largest real open interval containing 1 and find an antiderivative there. [3 marks]

(b) Evaluate its integral from 1 to 2 exactly. [2 marks]

3. Consider $\int (3x^2+1)/(x^3+x+2)\,dx$ for $x \geq 0$. [5 marks]

(a) Find an antiderivative and justify the logarithm domain. [3 marks]

(b) Evaluate the definite integral from 0 to 1. [2 marks]

4. Let $f(x) = (x^2 + 2x + 3)/[(x + 1)(x + 2)]$. [6 marks]

(a) Divide first, then give the partial fractions. [3 marks]

(b) Find an antiderivative and state the excluded inputs. [3 marks]

5. Consider $\int x^2 \cos x \, dx$. [6 marks]

(a) Use integration by parts twice to find an antiderivative. [4 marks]

(b) Evaluate the integral from 0 to π and explain why a negative answer is possible. [2 marks]

6. Let $I = \int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$. [4 marks]

(a) Derive a power-reduction expression for the integrand. [2 marks]

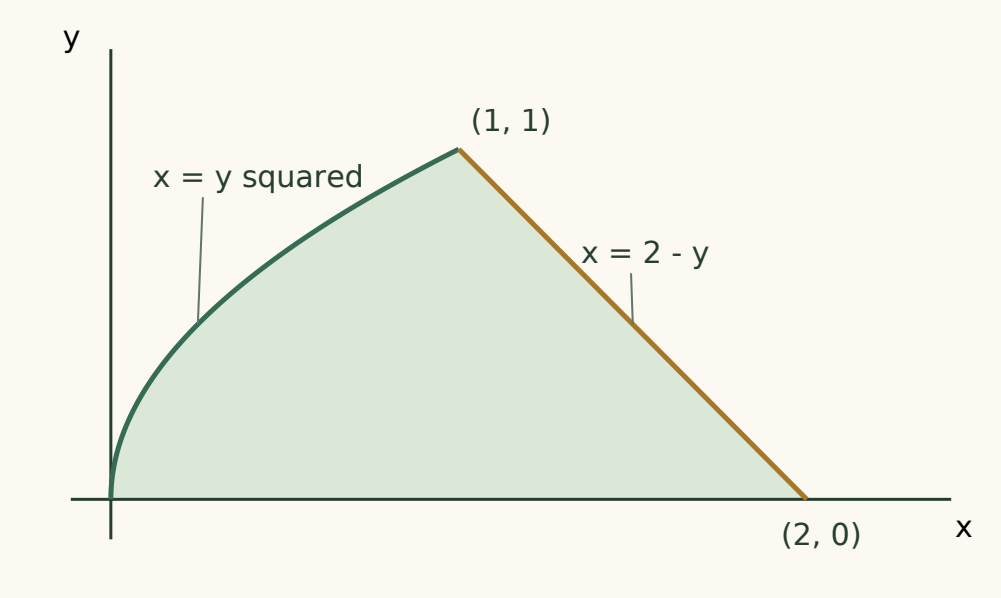
(b) Evaluate I exactly. [2 marks]

7. For $x > 0$, consider $\int \ln(2x) \, dx$. [6 marks]

(a) Find an antiderivative using parts. [3 marks]

(b) Find the average value of $\ln(2x)$ on $[1/2, e/2]$. [3 marks]

8. A region is defined by $0 \leq y \leq 1$ and $y^2 \leq x \leq 2 - y$. [6 marks]



(a) Find its area using horizontal slices. [3 marks]

(b) Find the volume when the region is rotated about the y axis. [3 marks]

9. Two force laws along $0 \le x \le 2$ metres are $F_1(x) = 3x + 1$ N and $F_2(x) = 3x^2$ N. [5 marks]

(a) Find the work done by each force over the full displacement. [3 marks]

(b) Does equal total work imply the forces are equal at every position? Use x=1 to justify your answer. [2 marks]

10. A tank starts with 30 L. Water enters at $q(t) = 20te^{-t^2}$ L/min for $t \ge 0$, with no outflow. [6 marks]

(a) Find the amount added by time T. [3 marks]

(b) Find when 8 L has been added, and state the tank amount then. [3 marks]