# Integration techniques and applications - practice test 5

Name: _____ Date: _____

10 questions | 58 marks | 75 minutes

Reasoning and applications across exact integrals, numerical approximation, changing regions and motion.

A scientific calculator may be used.

**1.** For all real x, $F'(x) = (2x + 3)/(x^2 + 3x + 5)$ and F(0)=0. [5 marks]

**(a)** Find F and justify that it is defined on the whole real line. [3 marks]

**(b)** Find F(2)-F(0). [2 marks]

**2.** Let $I = \int_0^1 (1 + x^2)^{-1}\,dx$ and $J = \int_0^1 (1 + 4x^2)^{-1}\,dx$. [5 marks]

**(a)** Evaluate I and J exactly. [3 marks]

**(b)** Without using decimal approximations, explain which is larger. [2 marks]

**3.** For $x > 0$, let $f(x) = (x^2 + x + 1)/[x(x + 1)^2]$. [6 marks]

**(a)** Find A, B and C in $f = A/x + B/(x + 1) + C/(x + 1)^2$. [3 marks]

**(b)** Evaluate the integral of f from 1 to 2. [3 marks]

**4.** Let $I = \int_0^{\pi/2} e^x \sin x \, dx$ and $J = \int_0^{\pi/2} e^x \cos x \, dx$. [6 marks]

**(a)** Use integration by parts to show that I+J equals $e^{\pi/2}$. [3 marks]

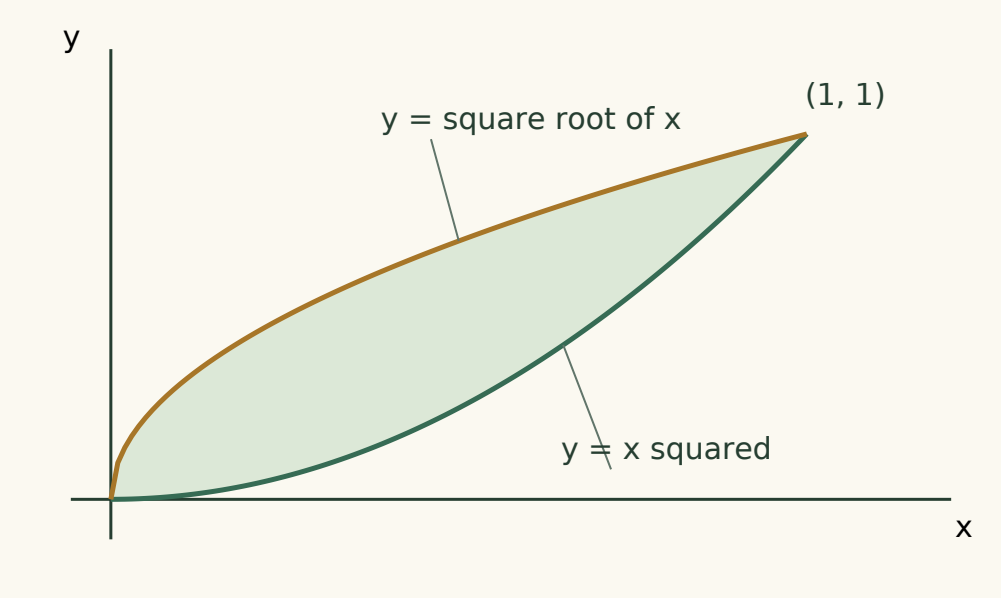
**(b)** Use parts on J, then solve for I. [3 marks]

**5.** The curve is $y = x^3 - x$ on $-2 \leq x \leq 2$. [6 marks]

**(a)** Find all zeros in the interval and the signed integral. [2 marks]

**(b)** Find the total geometric area between the curve and the axis. [4 marks]

**6.** A region lies between $y = \sqrt{x}$ and $y = x^2$ in the first quadrant. [9 marks]




**(a)** Find the intersection points and the bounded area. [3 marks]

**(b)** Find its volume when rotated about the x axis. [3 marks]

**(c)** Find its volume when rotated about the y axis, using horizontal slices. [3 marks]

**7.** A tank has horizontal cross-sectional area $A(h) = \pi(4 - h^2)\ \text{m}^2$ at height $h$ metres, for $0 \le h \le 2$. [5 marks]

**(a)** Find the volume below height h and the full capacity. [3 marks]

**(b)** At half the full height, what fraction of the full capacity is filled? [2 marks]

**8.** The table gives values of $f(x) = e^{-x^2}$. Use these values as supplied. [5 marks]

| x | f(x) |
|---|---|
| 0 | 1.0000 |
| 0.5 | 0.7788 |
| 1 | 0.3679 |

**(a)** Use the trapezium rule with two equal strips to estimate the integral from 0 to 1. [3 marks]

**(b)** A more accurate numerical value is 0.746824. Find the signed error T minus this value and the absolute error. [2 marks]

**9.** A periodic temperature model is $T(t) = 3 + 2\cos t + \sin^2 t$ degrees C for $0 \le t \le 2\pi$ hours. [5 marks]

**(a)** Find its exact average temperature over this interval. [3 marks]

**(b)** Explain why the average is above 3 degrees C despite the cosine term averaging to zero. [2 marks]

**10.** A particle moves on a straight line with velocity $v(t) = 3t^2 - 6t$ m/s for $0 \le t \le 3$. Its initial position is 2 m. [6 marks]

**(a)** Find its final position and average signed velocity. [3 marks]

**(b)** Find its total distance and average speed. [3 marks]