Implicit differentiation

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find dy/dx for $x^2 + 3y^2 = 12$. [2 marks]

$2x + 6yy' = 0$.

Divide by $6y$, where $y \neq 0$.

Answer: $-x/(3y)$

2. Find dy/dx for $x^3 + y^3 = 9$. [2 marks]

$3x^2 + 3y^2y' = 0$.

$y' = -x^2/y^2$, for $y \neq 0$.

Answer: $-x^2/y^2$

3. Find the tangent to $x^2 + y^2 = 10$ at $(1, 3)$. [2 marks]

The point satisfies $1 + 9 = 10$.

$y' = -x/y = -1/3$ at the point.

Answer: $y - 3 = -\frac{1}{3}(x - 1)$

4. Differentiate $x^2 + xy + y^2 = 7$ implicitly. [2 marks]

$2x + y + xy' + 2yy' = 0$.

Collect $y'(x + 2y) = -(2x + y)$.

Answer: $y' = -(2x + y)/(x + 2y)$

5. For $x^2 + y^2 = 9$, identify points where the tangent is vertical. [2 marks]

$y' = -x/y$ has a zero denominator at $y = 0$.

The circle then gives $x = \pm 3$; these are its left and right endpoints.

Answer: $(3, 0)$ and $(-3, 0)$

6. Find the gradient of $\sin y = x$ at $(0, 0)$. [2 marks]

$\cos y \, y' = 1$.

At $(0, 0)$, $\cos 0 = 1$, so $y' = 1$.

Answer: 1