Differential equations and initial conditions

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find the general solution of $y' = 4x^3$. [2 marks]

Integrate $4x^3$ to x^4.

Include an arbitrary constant.

Answer: $y = x^4 + C$

2. Solve $y' = \cos x$ with $y(0) = 2$. [2 marks]

$y = \sin x + C$.

$2 = 0 + C$, so $C = 2$.

Answer: $y = \sin x + 2$

3. Verify that $y = 3e^{-2x}$ solves $y' = -2y$. [2 marks]

$y' = -6e^{-2x}$.

$-2y = -6e^{-2x}$, equal to the derivative.

Answer: It does.

4. Does $y = x^2$ solve $y' = 2y$ for all real x? [2 marks]

The derivative is $2x$, but $2y = 2x^2$.

They are not equal for all x; equality at isolated points is insufficient.

Answer: No.

5. Solve $y' = 2/(x + 1)$ with $y(0) = 3$ on $x > -1$. [2 marks]

Integrate to $2\ln(x + 1) + C$ on the stated domain.

At $x = 0$, $C = 3$.

Answer: $y = 2\ln(x + 1) + 3$

6. An amount follows $P' = 0.04P$, $P(0) = 800$. Verify $P = 800e^{0.04t}$. [2 marks]

$P' = 32e^{0.04t} = 0.04P$.

$P(0) = 800$.

Answer: It satisfies both requirements.