Slope fields

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find slopes for $y' = xy$ at $(1, 2)$, $(2, -1)$ and $(0, 3)$. [2 marks]

Multiply the coordinates in each ordered pair.

Use each result as the local line gradient.

Answer: $2, -2, 0$

2. Where are the slopes zero for $y' = x + y$? [2 marks]

Set $x + y = 0$.

Solve for the line $y = -x$; this is not a constant equilibrium.

Answer: On $y = -x$.

3. For $y' = y(4 - y)$, identify constant equilibria. [2 marks]

Constant solutions need derivative zero.

$y(4 - y) = 0$ gives the two values.

Answer: $y = 0$ and $y = 4$.

4. For $y' = y(4 - y)$, state slope signs in $y < 0$, $0 < y < 4$ and $y > 4$. [2 marks]

Check the signs of both factors in each interval.

The sign pattern makes $y = 4$ stable for nearby positive values.

Answer: Negative, positive, negative.

5. For $y' = x^2$, what do segments at $(2, 1)$ and $(2, 5)$ have in common? [2 marks]

The equation depends only on x.

At $x = 2$, $x^2 = 4$ regardless of y.

Answer: Both have gradient 4.

6. For $y' = -2y$, describe whether a positive solution increases or decreases and whether it crosses the equilibrium. [2 marks]

For $y > 0$, $-2y < 0$.

$y = Ce^{-2x}$ with $C > 0$ stays positive for every finite x.

Answer: It decreases toward zero; the nonzero exponential solution does not cross zero.