Exponential and logistic models

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. An amount decays with half-life 5 hours and starts at 80 g. Find the exponential model. [2 marks]

Half-life means $e^{5k} = 1/2$.

$k = -\ln 2/5$ per hour.

Answer: $M = 80e^{-(\ln 2)t/5}$

2. $N = 120e^{0.08t}$, t in days. Find the time to reach 300. [2 marks]

Divide by 120 and take logarithms.

$0.08t = \ln 2.5$.

Answer: $t = \ln(2.5)/0.08 \approx 11.45$ days.

3. For $P' = 0.3P(1 - P/800)$ and $P(0) = 100$, find $P(t)$. [2 marks]

$A = (800 - 100)/100 = 7$.

Substitute r=0.3 and K=800.

Answer: $P = 800/(1 + 7e^{-0.3t})$

4. For the preceding logistic model, find the maximum growth rate. [2 marks]

The maximum occurs at half the carrying capacity.

$0.3(400)(1 - 400/800) = 60$.

Answer: 60 units/time at $P = 400$.

5. $P = 500/(1 + 9e^{-0.2t})$. Find the initial amount and long-term limit. [2 marks]

At zero the denominator is 10.

As t increases, the exponential term tends to zero.

Answer: $P(0) = 50$; limit 500.

6. Under $P' = 0.4P(1 - P/1000)$, state the predicted direction of change at $P = 1200$. [2 marks]

$P' = 0.4(1200)(1 - 1.2) = -96$.

The negative sign indicates decrease.

Answer: Decreasing, at 96 units/time.