Interpreting motion equations and simple harmonic motion

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. For $x = 4\cos(2t)$ m, find velocity and acceleration. [2 marks]

Apply the chain rule to each derivative.

The acceleration is $-4x$.

Answer: $v = -8\sin 2t$ m/s; $a = -16\cos 2t$ m/s^2.

2. Solve $x'' = -9x$, $x(0) = 0$, $v(0) = 6$. [2 marks]

Write $x = A\cos 3t + B\sin 3t$.

$A = 0$ and $3B = 6$, so $B = 2$.

Answer: $x = 2\sin 3t$

3. Find amplitude and period of $x = 3\cos t + 4\sin t$. [2 marks]

Amplitude is $\sqrt{3^2 + 4^2} = 5$.

The angular frequency is 1.

Answer: Amplitude 5; period 2π.

4. For $x = 5\cos 2t$, find the first positive time at equilibrium. [2 marks]

Equilibrium requires $\cos 2t = 0$.

The first positive solution is $2t = \pi/2$.

Answer: $\pi/4$

5. A particle has $a(t) = 2t$, $v(0) = 1$, $x(0) = 4$. Find its position. [2 marks]

Integrate to $v = t^2 + 1$.

Integrate again and apply the position condition.

Answer: $x = t^3/3 + t + 4$

6. A model has $x'' = -16(x - 2)$. State its equilibrium and period. [2 marks]

Displacement from equilibrium is $u = x - 2$.

Then $u'' = -16u$, so $\omega = 4$.

Answer: Equilibrium $x = 2$; period $\pi/2$.