# Rates of change and differential equations - Test 3

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Connect geometry, separation and rate models.

A scientific calculator may be used.

**1.** The relation $xy + y^2 = 6$ passes through (1, 2). [6 marks]

**(a)** Find $dy/dx$. [3 marks]

**(b)** Find the tangent at the stated point. [3 marks]

**2.** A conical tank has its vertex at the bottom and radius-to-height ratio $r = h/2$. Water enters at 3 $m^3$/min. At depth h=2 m: [6 marks]

**(a)** Write a relation suitable for differentiation. [3 marks]

**(b)** Find the water-depth rate in metres per minute. [3 marks]

**3.** The table shows values of a differentiable function satisfying $y' = 2x$. [6 marks]

Observed values

| x | y |
|---|---|
| 0 | 3 |
| 1 | 4 |
| 2 | 7 |
| 3 | 12 |

**(a)** Use the initial value in the table to find the function. [3 marks]

**(b)** Check whether all the table entries are consistent with this model. Does agreement with a finite table alone prove a differential equation for all real x? [3 marks]

**4.** Consider the slope field for $y' = y(y-2)$. [6 marks]

**(a)** Find slopes at $(0, 1)$ and $(1, 3)$. [3 marks]

**(b)** Find and classify constant equilibria, or explain why none exist. [3 marks]

**5.** Solve the initial-value problem $y' = 3/(x+1)$, $y(0)=2$, $x > -1$. [6 marks]

**(a)** Find $y(x)$. [3 marks]

**(b)** Find $y(e-1)$ and verify your derivative. [3 marks]

**6.** Consider $y' = x/y$, $y(0)=2$. [6 marks]

**(a)** Find the solution and its interval containing the initial point. [3 marks]

**(b)** Identify any constant solution lost during separation, or explain why there is none. [3 marks]

**7.** An object cools according to $T' = -0.2(T - 20)$, with $T(0) = 80$. Temperature is in degrees Celsius and time in minutes. [6 marks]

**(a)** Find the temperature model. [3 marks]

**(b)** Find when the temperature first reaches 40 degrees Celsius. [3 marks]

**8.** A population satisfies $P' = 0.3P(1 - P/800)$ with $P(0) = 200$. Here t is in days, P is in individuals, and t is at least zero. [6 marks]

**(a)** Find $P(t)$ and the carrying capacity. [3 marks]

**(b)** Find the global largest possible positive growth rate and when this trajectory reaches its associated amount. [3 marks]

**9.** A displacement model is $x'' = -x$, $x(0)=3$, $v(0)=4$. Here t is measured in seconds and displacement x in metres. [6 marks]

**(a)** Find or verify the displacement and its derivative relation. [3 marks]

**(b)** Find the amplitude and period. [3 marks]

**10.** A logistic rate is $P' = 0.5P(1 - P/400)$. A student says growth is greatest at P=400. [6 marks]

**(a)** Identify and correct the error. [3 marks]

**(b)** Calculate the greatest possible growth rate. [3 marks]