Rates of change and differential equations - Test 6

Name: _____ Date: _____

10 questions | 63 marks | 75 minutes

Combine methods and critique common modelling errors.

A scientific calculator may be used.

1. The curve $x^2y + xy^2 = 6$ passes through (1, 2). [6 marks]

(a) Find dy/dx. [3 marks]

(b) Find the tangent at the stated point. [3 marks]

2. A rectangle has side x and perimeter 30 cm. Its x side increases at 1 cm/s. At x=4 cm: [6 marks]

(a) Write a relation suitable for differentiation. [3 marks]

(b) Find the area rate in square centimetres per second. [3 marks]

3. Two proposed solutions are $f(x) = 2e^x$ and $g(x) = e^{2x}$. The initial-value problem is $y' = 2y$, $y(0) = 2$. [6 marks]

(a) Explain which requirement each proposed function fails. [3 marks]

(b) Find the correct solution and verify both requirements. [3 marks]

4. The equation is $y' = 1 - y^2$. A numerical sketch begins at $y(0) = 0$. [6 marks]

(a) Identify and classify the constant equilibria using the slope signs. [3 marks]

(b) A sketch crosses y=1 with positive slope. Explain the error, and verify the proposed exact solution $y = (e^{2x} - 1)/(e^{2x} + 1)$. [3 marks]

5. A particle has acceleration $a(t) = 6t - 6$ m/s^2, $v(0) = 0$ and $x(0) = 5$ m for $0 \le t \le 3$ s. [6 marks]

(a) Find its velocity and position. [3 marks]

(b) Find its displacement and total distance during the three seconds. [3 marks]

6. Consider $y' = y/(x+1)$, $y(0)$=-2, $x > -1$. [6 marks]

(a) Find the solution and its interval containing the initial point. [3 marks]

(b) Identify any constant solution lost during separation, or explain why there is none. [3 marks]

7. A reservoir contains V litres and satisfies $V' = 4 - 0.1V$, $V(0) = 10$, with t in minutes. The inflow is 4 L/min and the outflow rate is 0.1 V L/min. [6 marks]

(a) Find the amount model. [3 marks]

(b) Find the long-term amount and the initial rate. [3 marks]

8. A population satisfies $P' = 0.2P(1 - P/900)$ with $P(0) = 600$. Here t is in days, P is in individuals, and t is at least zero. [9 marks]

(a) Find $P(t)$ and the carrying capacity. [3 marks]

(b) Find the global largest possible positive growth rate and when this trajectory reaches its associated amount. [3 marks]

(c) For this initial population, find the greatest growth rate for t at least zero. [3 marks]

9. A displacement model is $x = 2\cos 3t - 2\sin 3t$. Here t is measured in seconds and displacement x in metres. [6 marks]

(a) Find or verify the displacement and its derivative relation. [3 marks]

(b) Find the amplitude and maximum speed. [3 marks]

10. For $y' = x - y$, a student calls $y = x$ an equilibrium because its field slopes vanish. [6 marks]

(a) Identify and correct the error. [3 marks]

(b) Prove that no constant equilibrium exists. [3 marks]